Reading Group
Bordered knot Floer homology

Winter semester 2024

17/10 | 1 | Introduction (Claudius) [1, Section 1]+[8, Sections 1+2]

- discuss overall strategy to define bordered knot Floer homology and goals for the course
- Kauffman states and the Alexander polynomial
- Kauffman states for tangles
- *homework: read [8, Section 4] and compute the Kauffman states for all basic tangles in the trefoil knot + do the gluing*

24/10 | 2 | The bordered algebras (Cara) [8, Section 5.1]+[1, Sections 2.1+2.2]

- discuss homework
- compute $\mathcal{C}_0(n)$ and $\mathcal{C}(n)$ for $1 \leqslant n \leqslant 3$
- compare with $\mathcal{B}_0(m,k)$, $\mathcal{B}(m,k)$, $\mathcal{A}(m,k,M)$ and $\mathcal{A}'(m,k,M)$
- *homework: read [7, Sections 3.1+3.2] and compute the morphism spaces for* $\mathcal{C}(2) \subseteq \mathcal{B}(4,2)$

24/10 | 3 | Algebraic preliminaries I (Claudius) [8, Section 3]+[7, Section 2.3+2.4]

- the dg category of chain complexes over categories, with and without curvature
- special case I: the category of type D structures
- *homework: write down some chain complex over* $\mathcal{C}(2)$ *with at least five generators and at least five non-zero components of the differential. Then check that* $d^2 = 0$.

07/11 | 4 | Algebraic preliminaries II (Claudius) [8, Section 7.1]+[1, Section 5.1+5.2]

- discuss homework from lecture 2 (lecture 3 only if there are questions)
- special case II: the category type A structures
- pairing type A and type D structures
- special case III: the categories of type AD (DD, AA, DA) bimodules
- *homework: write down some type A structure over* $\mathcal{C}(2)$ *with at least five generators and at least five non-zero components of the differential. Then check that the structure relation holds. Then do the pairing with the type D structure from previous homework.*

07/11 | 5 | Bimodule for a maximum and pairing I (Chen) [8, Section 7.1]+[1, Section 5.1+5.2]

- recap of the construction of the knot invariant

07/11 | ★ | Léo Schelstraete: Oberseminar talk at 4pm

21/11 | ★ | postponed due to illness

26/11 | 6 | Bimodule for a maximum and pairing II (Chen) [8, Section 7.1]+[1, Section 5.1+5.2]

- compute bimodules for local maximums for different number of strands
- compute bimodules for various local maximums

26/11 | 7 | Bimodule for a crossing I (Subhankar) [8, Section 7.4]+[1, Section 3]

- positive crossing on two strands

12/12 | 8 | Bimodule for a crossing II (Chen) [1, Section 3] and [2, Def. 3.3 and Section 12.1]

- positive crossing on two strands with curvature
- positive crossing on many strands with curvature
- compute the invariant for the one crossing tangle needed for the computation for the right-handed trefoil

12/12 | 9 | Bimodule for a crossing III (Claudius) [1, Sec. 3] and [2, Def. 3.3 and Sec. 12.1]

- Computation for the trefoil knot
- comparison positive/negative crossings
- *homework: finish the computation for the trefoil knot except for the minima. More explicitly, starting from the type D structure of the two maxima with matchings (1,4) and (2,3), compute the type D structures over $\mathcal{C}(2)$ obtained by adding three crossings between the first and second strand. At each step, check that the type D structure indeed satisfies the d^2-relation.*

19/12 | 10 | More computations (Claudius) [13, Section 2.2]

- Reidemeister II computation
- Clean-up and Cancellation lemmas
- *homework 1: For all positive integers n, compute the type D structure corresponding to n positive twists between strands 1 and 2 (continuing the trefoil computation).*
- *homework 2: Complete the proofs of the clean-up and cancellation lemmas.*

09/01 | 11 | Local minima (Claudius) [1, Section 7]

- local minimum bimodule
- Compute the knot Floer chain complex for the torus knots $T_{2,2n+1}$
- The type D structure for the $(2,-3)$-pretzel tangle

16/01 | 12 | General Reidemeister II move on bimodules (Chen)
[1, Section 3.1, Proposition 3.4], [7, Sections 3.7, 3.8, Lemma 6.3]

- The canonical bimodule (proof of invertibility)
- RM II invariance (pairing type AD with DD).

23/01 | 13 | The Alexander grading and the invariant τ (Cara)
[12, Sections 11-13], [1, Sections 2.2 and page 504]

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References

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[3] P. S. Ozsváth, Z. Szabó: Algebras with matchings and link Floer homology. ArXiv preprint 2004.07309

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[8] A. Alfieri, J. Van Dyke: An introduction to knot Floer homology and curved bordered algebras, Period Math Hung 80, 211–236 (2020). Available at `https://doi.org/10.1007/s10998-020-00311-8`

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[12] P. S. Ozsváth, Z. Szabó: An introduction to Heegaard Floer homology. `https://web.math.princeton.edu/~petero/Introduction.pdf`

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