updated 3rd April 2019

MATH 101:207 Integral Calculus with Applications to Physical Sciences and Engineering

lecture summaries

spring term 2018/2019

MWF 16:00-16:50 (Buch A104)

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Integration

W 02/01 | **1** | **Introduction to integration**

- The integral $\int_a^b e^x \, \mathrm{d}x$ (1.1)
- Geometric sum formula (equation 1.1.3)
- Quick review: summation notation (1.1.2)

F 04/01 | **2** | **Definite integrals**

- Definition: Riemann sums and the definite integral (0.4)
- Geometric interpretation of integrals as signed area (1.1.3)

M 07/01 | **3** | **Basic computations**

- Computing integrals using areas of known shapes (1.1.4)
- Basic properties of the definite integral (1.2)
- Computing integrals of even and odd functions (1.1.4)

W 09/01 | **4** | **The fundamental theorem of calculus I**

- A physical interpretation of integrals (1.1.5)
- The fundamental theorem of calculus (FTC) version 1 (theorem 1.3.1, part 2)
- Examples: reading differentiation tables “upside-down” (examples 1.3.11ff)

F 11/01 | **5** | **The fundamental theorem of calculus II**

- More examples: reading differentiation tables “upside-down” (examples 1.3.11ff)
- The fundamental theorem of calculus (FTC) version 2 (theorem 1.3.1, part 1)
- (FTC version 1) $\Leftrightarrow$ (FTC version 2); proof of (FTC version 2)

M 14/01 | **6** | **The fundamental theorem of calculus III**

- More examples about how to use the FTC: in particular $\int \frac{1}{x} \, \mathrm{d}x = \ln|x|$ (examples 1.3.13)
- Examples about how *not* to use the FTC: in particular, checking domains and continuity!

W 16/01 | 7 | Substitution rule for indefinite integral

- A little test
- Revision: Linearity of integration and differentiation (1.4)
- Substitution rule from chain rule for the indefinite integral; examples (theorem 1.4.2ff)

F 18/01 | 8 | Substitution rule for definite integral

- More examples on the substitution rule for definite integrals (theorem 1.4.2ff)
- Substitution rule for definite integral (theorem 1.4.6)
- Desmos presentation: Area between curves (see announcement on Canvas)

M 21/01 | 9 | Area between curves

- Example: total area between two curves that intersect multiple times (see announcement on Canvas)
- Example: integrate in y-direction (1.5)

W 23/01 | 10 | Volumes

- Volume = integrated area (1.6)
- Examples: volume of cone, sphere, pyramid, torus (1.6)
- Exercise: cones of arbitrary shapes

F 25/01 | 11 | Volumes (continued) and averages

- Computation for torus (continued)
- Volume of the perpendicular intersection of two cylinders of unit radius
- Volume of a solid obtained by rotation about another line than simply $x = 0$ or $y = 0$
- Definition and example: the average of a function (2.2)

M 28/01 | 12 | Work

- Definitions and basic examples, in particular: Newton's second law and Hooke's law (2.1.2)
- Examples: pumping water out of a conical and a spherical tank (2.1.4)

W 30/01 | 13 | Integration by parts

- Theorem and many examples (1.7)

F 01/02 | 14 | Trigonometric integrals with powers of sin and cos

- General algorithm and many examples (1.8, 1.8.1)

M 04/02 | 15 | Trigonometric integrals with powers of sec and tan

- General algorithm and many examples (1.8.2)

W 06/02 | 16 | Trigonometric substitutions

- The three prototypical examples: $\int\sqrt{a^2-x^2}\mathrm{d}x$, $\int\sqrt{x^2-a^2}\mathrm{d}x$ ad $\int\sqrt{x^2+a^2}\mathrm{d}x$ (1.9)
- Undoing the substitution after successful integration (1.9.3, 1.9.5, 1.9.6)
- Importance of pausing for a moment before ploughing on (example 1.9.4)

F 08/02 | 17 | Trigonometric substitutions (continued) and partial fractions

- Completing the square before trigonometric substitutions (example 1.9.7)
- Basic principle of integrating rational functions (1.10)

M 11/02 | 18 | Partial fractions (continued)

- Factorization and partial fraction decomposition with linear and quadratic terms (1.10)

W 13/02 | 19 | Review session

- Partial fractions, FTC, Riemann sums

F 15/02 | ★ | Midterm exam

18–22/02 | ★ | Midterm break

Applications of integration

M 25/02 | 20 | Numerical integration

- Approximations via Taylor polynomials
- Approximations via midpoint rule, trapezoid rule and Simpson's rule (1.11.1–1.11.3)

W 27/02 | 21 | Error terms in numerical integration+improper integrals I

- Explanation of Simpson's Rule (1.11.3)
- Comparison of error terms for midpoint rule, trapezoid rule and Simpson's rule (1.11.4)
- Improper integrals with unbounded limits of integration (1.12.1)

F 01/03 | 22 | Improper integrals II

- More examples: improper integrals with unbounded limits of integration (1.12.2)
- Improper integrals with unbounded functions (1.12.1)

M 04/03 | 23 | Convergence tests and center of gravity (1D)

- More examples: improper integrals with unbounded functions (1.12.2)
- Convergence tests for improper integrals (1.12.3)
- Centres of mass: 1-dimensional case (2.3.1)

W 06/03 | 24 | Center of gravity (2D) and differential equations

- Example: centre of mass of a cone (2.3.1)
- Centres of mass: 2-dimensional case (example: inverted parabola) (2.3.1)
- A primer on differential equations (2.4)

Sequences and Series

F 08/03 | 25 | Separable differential equations and sequences

- Separable differential equations (2.4.1)
- Definition and examples: sequences, convergence, divergence (3.1)

M 11/03 | 26 | Sequences and series

- General principles and arithmetics for limits of sequences (3.1)
- Exam revision strategies (see the revision checklist)
- New from old: series and their limits (3.2)

W 13/03 | 27 | Lots of series

- Geometric series, telescoping series, harmonic series (3.2+3.3.9/warning 3.3.3)

F 15/03 | 28 | Divergence test and integral test

- Main examples: $\sum_{n=1}^{\infty} \frac{1}{n}$ (harmonic series) and $\sum_{n=1}^{\infty} \frac{1}{n^2}$ (3.3)

M 18/03 | 29 | Integral test with remainder and comparison test

- Main examples: $\sum_{n=1}^{\infty} \frac{1}{n^p}$ (most important one), $\sum_{n=1}^{\infty} \frac{1}{n \cdot (\log(n))^p}$, $\sum_{n=1}^{\infty} \frac{\log(n)}{n}$ (3.3.2+3.3.3)

W 20/03 | 30 | Comparison test

- Comparison and limit comparison test: more examples of the form $\sum_{n=1}^{\infty} R(n)$ for some rational function $R(n)$ (3.3.3)

F 22/03 | 31 | Alternating series and remainders (lecture given by John Enns)
M 25/03 | 32 | Absolute/conditional convergence, ratio test (lecture given by John Enns)
W 27/03 | 33 | Power series I

- Definition, formula and examples: convergence radius and interval of convergence (3.5.1)

F 29/03 | 34 | Power series II

- Exam preparation: summary of convergence tests (3.3.6)
- Differentiation and integration of power series (theorem 3.5.13)
- Examples: differentiating and integrating the geometric series (examples 3.5.20+3.5.21)

M 01/04 | 35 | Taylor series

- Definition and examples: Taylor and Maclaurin series (theorem 3.6.5)
- Maclaurin series: new from old via substitution (example 3.6.10)

W 03/04 | 36 | Exam-type questions for Taylor series

- Maclaurin series and limits (3.6.4)
- Maclaurin series and improper integrals

Good luck with the final exam!