MATH 100:101 Differential Calculus
with Applications to Physical Sciences and Engineering

lecture summaries

winter term 2019/2020
MWF 10:00–10:50 (LSK 200)
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Limits

W 04/09 | 1 | Introduction to differential calculus: Newton's apple

- Observation (Newton 1666): Apples fall down! (1.2)
- What is velocity?
- Linear motion and average velocities
- Back to the apple: instantaneous velocity

F 06/09 | 2 | A first look at limits

- Limits as a method to extend functions to where they are not defined (definition 1.3.3)
- Limits do not always exist (example 1.3.5)
- A limit exists if both one-sided limits exist and agree with each other (theorem 1.3.8)

M 09/09 | 3 | Arithmetic of limits

- $\pm\infty$ as special DNE(=does-not-exist) values of limits (definition 1.3.11)
- Limit laws (sums, differences, products, quotients, powers, roots, rational functions) (1.4)
- Behaviours of $\lim\limits_{x\to a}\frac{f(x)}{g(x)}$, where $g(a)=0$ (1.4)

W 11/09 | 4 | Squeezing theorem and limits at ∞

- Calculating limits of fractions by multiplying with conjugate (Example 1.4.16)
- Squeezing theorem and examples (1.4)
- Limits at infinity (main trick: factor out the dominant term) (1.5)

F 13/09 | 5 | Continuity (lecture by Jamie Juul)

- More examples of limits at infinity: limits of type ∞/∞ and $\infty-\infty$ (1.5)
- Definition: the absolute value $|x|$ of a real number x (1.5)
- Definition and discussion: continuity (Example 1.6.4)

M 16/09 | **6** | **The IVT and a first look at derivatives**

- Intermediate Value Theorem (IVT) (1.6)
- Newton's example revisited: slope of tangents = limit of slope of secants (1.1–1.2)

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derivatives

W 18/09 | **7** | **Tangents and derivatives**

- Definition: the derivative of a function at a point (2.2)
- Calculation of linear and quadratic functions (2.2)
- Computing tangents (2.3)

F 20/09 | **8** | **Arithmetic of derivatives I: sums and differences**

- 3Blue1Brown youtube channel
- Derivative as a function and all sorts of notation (2.2)
- Theorem: $f(x)$ is differentiable $\Rightarrow$ $f(x)$ is continuous (2.2)
- Derivatives of sums (2.4)

M 23/09 | **9** | **Arithmetic of derivatives II: products and quotients**

- Derivatives of products (2.4)
- $\frac{\mathrm{d}}{\mathrm{d}x}(x^a) = a \cdot x^{a-1}$ for any rational numbers a (2.6)
- The quotient rule as a special case of the product rule (2.4, 2.5, 2.6)

W 25/09 | **10** | **Arithmetic of derivatives III: exponential and trigonometric functions**

- Euler's number and why $y = e^x$ is such a special function (2.7)
- Derivatives of trigonometric functions sin, cos, tan, cot, csc, sec (2.8)

F 27/09 | **★** | **UBC Climate Strike**

M 30/09 | **11** | **Arithmetic of derivatives IV: chain rule**

- The chain rule, why it is true and plenty of examples (2.9)

W 02/10 | **12** | **Arithmetic of derivatives V: inverse functions and logarithm**

- Absolute values and the chain rule
- Logarithms and their derivatives (2.10)
- Inverse functions (0.6)

F 04/10 | 13 | Arithmetic of derivatives VI: Inverse trigonometric functions

- Revision strategies
- Inverse trigonometric functions and their derivatives (2.12)

M 07/10 | 14 | Implicit differentiation I

- More examples for inverse trigonometric functions (2.12)
- Logarithmic differentiation (differentiate $\log|f(x)|$ instead of $f(x)$) (2.10)
- Implicit differentiation: Finding tangents to curves in the plane (2.11)

W 09/10 | 15 | Implicit differentiation II

- Finding tangents to curves in the plane (2.11)
- More examples for logarithmic differentiation (2.10)
- Higher derivatives (2.14)

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applications of derivatives

F 11/10 | 16 | Rates of change

- 5-minute test (past exam question on IVT)
- Newton's example revisited: velocity and acceleration (3.1)

M 14/10 | ★ | Thanksgiving holiday

W 16/10 | 17 | Exponential decay I

- 10-minute test (gluing two functions together to geth differentiable function)
- Exponential decay: Carbon dating (3.3.1)

F 18/10 | ★ | midterm exam

M 21/10 | 18 | Exponential decay II

- Exponential decay: Carbon dating (continued) (3.3.1)
- Exponential decay: Newtons law of cooling (3.3.2)

W 23/10 | 19 | Exponential growth + related rates

- Exponential decay: Newtons law of cooling (continued) (3.3.2)
- Exponential growth: population growth (3.3.3)
- Related rates: problem session (exercises) (3.2)

F 25/10 | 20 | Linear approximations and towards Taylor polynomials

- Related rates: problem session (continued) (exercises) (3.2)
- Linear and quadratic approximations of $\sin(x)$ and $\cos(x)$ at $x_0 = 0$ (3.4.1-3.4.3)

M 28/10 | 21 | Taylor polynomials

- Maclaurin's polynomial: approximations of differentiable functions at $x_0 = 0$ (3.4.4)
- Example: Maclaurin's polynomials of $\sin(x)$ and $\cos(x)$
- Definition: Taylor polynomial as shifted Maclaurin's polynomial (3.4.4)
- Example: Taylor's polynomial for $\log x$ at $x_0 = 1$ (3.4.5)

W 30/10 | 22 | Taylor's formula and remainders

- Summation notation, Fibonacci numbers and lots of examples (3.4.3–3.4.7)
- Remainder formula for Taylor polynomials (3.4.8)

F 01/11 | 23 | More on remainder terms + Maxima and minima I

- Exercise: $e \approx 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots + \frac{1}{k!}$ with error estimate (3.4.8)
- Why do Taylor polynomials for e^x and $\log(x)$ behave differently? (non-examinable)
- Theorem: If the maximum value of $f(x)$ is $f(c)$ and $f'(c)$ exists, then $f'(c) = 0$. (3.5.1)

M 04/11 | 24 | Maxima and minima II

- Methods for finding local maxima and minima (3.5.1)
- Using second derivatives to determine the type of extrema (theorem 3.5.5)
- Critical points, singular points and boundary points (3.5.1)

W 06/11 | 25 | Mean value theorem (lecture by Keegan Boyle)

- Theorem: global extrema on closed intervals (3.5.2)
- Le théorème de Rolle and the mean value theorem (MVT) (2.13)

F 08/11 | 26 | More applications of MVT (guest lecture by Zach Goldthorpe, teaching student in MATH599)

- Various applications of the MVT
- MVT as the origin of the error term for Taylor polynomials

M 11/11 | ★ | Remembrance day

W 13/11 | 27 | Curve sketching I (first derivative)

- Corollary: $f'(x) = 0$, $f'(x) < 0$, $f'(x) > 0$ for all $x \in (a, b)$ implies that $f(x)$ is constant, decreasing, increasing on (a, b), respectively. (2.13)
- Corollary: $f'(x) = g'(x)$, implies $f(x) = g(x) + c$ for some constant c (2.13)
- Using the first derivative for graph sketching (example 3.6.2)

F 15/11 | 28 | Curve sketching II (second derivatives + asymptotes)

- Concavity and convexity (3.6.3)
- example: sketching rational functions (cp. example 3.6.14)
- example: domains need not be all of $\mathbb{R}$!

M 18/11 | 29 | Curve sketching III (symmetries)

- Even and odd functions: definition and geometric interpretation (3.6.4)
- Exercise: Taylor polynomials of even/odd functions
- Periodicity: definition, geometric interpretation and examples (3.6.4)
- Bonus: All "nice" functions can be written as sums of $\sin(x)$ and $\cos(x)$ (Fourier on youtube)

W 20/11 | 30 | L'Hôpital's rule I

- L'Hôpital's rule for "$\frac{0}{0}$" and "$\frac{\pm\infty}{\pm\infty}$" (3.7.1–2b)
- Variations of l'Hôpital's rule: type "$0 \cdot \infty$" and type "$\infty - \infty$" (3.7.2c–d)
- Geometric explanation of l'Hôpital's rule using tangents

F 22/11 | 31 | L'Hôpital's rule II + Optimisation problems I

- Even more variations of l'Hôpital's rule: type "1^∞", type "0^0" and type "∞^0" (3.7.2e–g)
- Example: optimisation of the volume of a rectangular box (example 3.5.16)

M 25/11 | 32 | Optimisation problems II

- Example: minimal distance between a point and a line (example 3.5.18)
- Exercise: Largest right circular cylinder inscribed in a sphere
- Example: taking a photo of the Statue of Liberty (example 3.5.24)

towards integral calculus

W 27/11 | 33 | A first glance at antiderivatives (lecture notes)

- Definition and examples: antiderivatives (4.1)
- Theorem: antiderivatives are unique up to adding a constant (4.1)
- Guessing antiderivatives by reading differentiation tables uʍop-ǝpᴉsdn (4.1)
- Application of antiderivatives: review of the exponential differential equation (4.1.6)

F 29/11 | 34 | Mock exam

- Mock exam (and the solutions)

Happy exam revision and all the luck you need!