MATH 100:102 Differential Calculus with Applications to Physical Sciences and Engineering

lecture summaries

winter term 2018/2019

Tuesdays 8:00–9:30 (Math 100) | Thursdays 8:00–9:30 (Math 100)

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functions and limits

Th 06/09 | 1 | Introduction to differential calculus

- What is differential calculus?
- linear functions and their slopes
- functions and their graphs (0.4)
- proposition: When is a subset of $\mathbb{R}^2$ the graph of a function? (a.k.a. vertical line test)
- tangents and velocity (Newton's example) (1.1-1.2)

Tu 11/09 | 2 | Limits

- Newton's example (continued) (1.2)
- limits as a method to extend functions to where they are not defined (1.3)
- one-sided limits (1.3)

Th 13/09 | 3 | Arithmetic of limits

- limit laws (sums, differences, products, quotients, powers, roots, rational functions) (1.4)
- Behaviours of $\lim\limits_{x\to a} \frac{f(x)}{g(x)}$, where $g(a) = 0$ (1.4)
- Squeezing theorem and examples (1.4)
- Limits at infinity (definition only) (1.5)

Tu 18/09 | 4 | limits at ∞ and continuity

- Limits at infinity (main trick: factor out the dominant term) (1.5)
- definition: the absolute value $|x|$ of a real number x (1.5)
- definition and discussion: continuity (1.6)
- Intermediate Value Theorem (IVT) (1.6)

derivatives

Th 20/09 | 5 | tangents and derivatives

- Newton's example revisited (2.1)
- definition: the derivative of a function at a point and all sorts of notation (2.2)
- calculation of various derivatives: linear functions, quadratic functions, $\frac{1}{x}$, $|x|$, $x^{1/3}$ (2.2)
- continuity and differentiation (example) (2.2)

Tu 25/09 | 6 | Arithmetic of derivatives I

- continuity and differentiation (theorem) (2.2)
- computing tangents (2.3)
- Derivatives of sums, products, integer powers (2.4, 2.6)

Th 27/09 | 7 | Arithmetic of derivatives II

- $\frac{\mathrm{d}}{\mathrm{d}x}(x^a) = a \cdot x^{a-1}$ for any rational numbers a (2.6)
- The quotient rule as a special case of the product rule (2.4, 2.5, 2.6)
- Derivatives of trigonometric functions (with proofs) (2.8)

Tu 02/10 | 8 | Arithmetic of derivatives III

- The chain rule (2.9)
- Derivatives of exponentials (2.7)
- Inverse functions (0.6)

Th 04/10 | 9 | Implicit differentiation

- Logarithms and their derivatives (2.10)
- Logarithmic differentiation (differentiate $\log|f(x)|$ instead of $f(x)$) (2.10)
- Implicit differentiation (2.11)
- finding tangents to curves in the plane (2.11)

Tu 09/10 | 10 | Inverse trigonometric functions

- 5-minute test: (past exam question on implicit differentiation) (see here or here)
- Inverse trigonometric functions and their derivatives (2.12)
- general epistemological remarks

applications of derivatives

Th 11/10 | 11 | Rates of change I

- Newton's example revisited: velocity and acceleration (3.1)
- Exponential decay: Carbon dating (3.3.1)

Tu 16/10 | 12 | Rates of change II

- Exponential decay: Newton's law of cooling (3.3.2)
- Exponential growth: Population growth (3.3.3)

Th 18/10 | ★ | mid-term exam

Tu 23/10 | 13 | Problem solving strategies

- discussion: mid-term exam question 5 (group A)/question 4 (group B)
- problem session on related rates (problem sheet) (3.2)

Th 25/10 | 14 | Taylor polynomials

- example: linear approximation of $\sin x$ at $x_0 = 0$ (3.4.2)
- example: quadratic approximation of $\cos x$ at $x_0 = 0$ (3.4.3)
- polynomial approximations of differentiable functions at $x_0 = 0$ (Maclaurin's polynomial) (3.4.4)
- exercise: $e \approx 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots + \frac{1}{k!}$.

Tu 30/10 | 15 | Taylor's formula and remainders

- definition: Taylor polynomial as shifted Maclaurin's polynomial (3.4.4)
- summation notation (3.4.3)
- example: Taylor's polynomial for $\log x$ at $x_0 = 1$ (3.4.5)

Th 01/11 | 16 | Maxima and minima

- (rather artificial) applications of Taylor polynomials: error estimates (3.4.6–3.4.7)
- methods for finding local maxima and minima (3.5.1)
- using second derivatives to determine the type of extrema (theorem 3.5.5)
- critical points, singular points and boundary points (3.5.1)

Tu 06/11 | 17 | Mean value theorem

- theorem: global extrema on closed intervals (3.5.2)
- Le théorème de Rolle and the mean value theorem (MVT) (2.13)
- application: The equation $2x - 1 = \sin(x)$ has a unique solution. (example 2.13.3)

Th 08/11 | 18 | More applications of MVT and curve sketching I (asymptotes)

- corollary: $f'(x) = 0$, $f'(x) < 0$, $f'(x) > 0$ for all $x \in (a, b)$ implies that $f(x)$ is constant, decreasing, increasing on (a, b), respectively. (2.13)
- corollary: $f'(x) = g'(x)$, implies $f(x) = g(x) + c$ for some constant c (2.13)
- corollary. Remainder in Taylor polynomials with (non-examinable) proof using complete induction (3.4.8)
- example: sketching rational functions (3.6)

Tu 13/11 | 19 | Curve sketching II (first and second derivatives)

- Using the first derivative for graph sketching (example 3.6.2)
- Concavity and convexity (3.6.3)
- exercise: sketching rational functions (cp. example 3.6.14)
- example: sketching functions with singular points (example 3.6.15)

Th 15/11 | 20 | Curve sketching III (symmetries)

- Even and odd functions: definition and geometric interpretation (3.6.4)
- exercise: Taylor polynomials of even/odd functions
- periodicity: definition and geometric interpretation (3.6.4)
- example: Sketch $y = \cos^2(x) + 1$
- remark: All periodic functions can be written as sums of sin and cos (google Fourier series)

Tu 20/11 | 21 | Optimisation problems [lecture given by Liam Watson]

- example: optimisation of the volume of a rectangular box (example 3.5.16)
- example: minimal distance between a point and a line in the Euclidean plane (example 3.5.18)

Th 22/11 | 22 | L'Hôpital's rule

- theorem: existence of extrema of continuous functions with asymptotes at the boundary of the domain (theorem 3.5.17)
- exercise: minimal distance between a point and a curve in the Euclidean plane (example 3.5.19)
- Translating a text question into a math question (3.5.3)
- l'Hôpital's rule for "$\frac{0}{0}$" and examples (3.7.1)
- warning: $\lim_{x\to a} \frac{f'(x)}{g'(x)} = DNE$ does NOT imply $\lim_{x\to a} \frac{f(x)}{g(x)} = DNE$ (warnings 3.7.7–3.7.9)

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towards integral calculus

Tu 27/11 | 23 | Special cases of l'Hôpital's rule and a first glance at antiderivatives

- l'Hôpital's rule for "$\frac{\pm\infty}{\pm\infty}$" and examples (3.7.2a–b)
- more variations of l'Hôpital's rule: type "$0 \cdot \infty$" and type "$\infty - \infty$" (3.7.2c–d)
- even more variations of l'Hôpital's rule: type "1^∞", type "0^0" and type "∞^0" (3.7.2e–g)
- definition and examples: antiderivatives (4.1)
- theorem: antiderivatives are unique up to a constant (4.1)

Th 29/11 | 24 | More on antiderivatives and review

- guessing antiderivatives by reading differentiation tables uʍop-ǝpᴉsdn (4.1)
- application of antiderivatives: velocity and position (4.1.5)
- application of antiderivatives: review of the exponential differential equation (4.1.6)
- A physicist's proof of the volume formula of a cone (for an alternative proof, see 4.1.7)

Good luck with the exam!