Khovanov and Heegaard Floer theories through the lens of immersed curves II

Claudius Zibrowius

based on the work of many people,
but in particular on joint work with
Artem Kotelskiy and Liam Watson

University of British Columbia

Towards immersed curves in Heegaard Floer theories

for closed 3-dimensional objects

- Heegaard Floer homology [Ozsváth-Szabó]
- Knot and link Floer homology [Ozsváth-Szabó, Rasmussen]

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immersed curve invariants

$||$

geometric reformulations of these relative theories in special cases

Immersed curves for 3-manifolds with torus boundary

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$$\begin{pmatrix} \text{3-dim. manifold } M \\ \text{with torus boundary} \end{pmatrix} \longmapsto \begin{pmatrix} \text{immersed curves* } \widehat{\mathrm{HF}}(M) \\ \text{on } \partial M \text{ minus a basepoint} \end{pmatrix}$$

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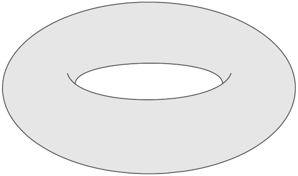

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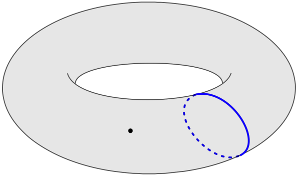

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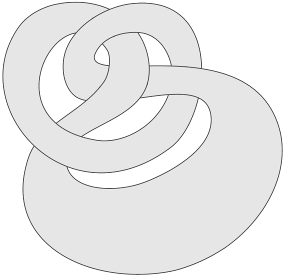

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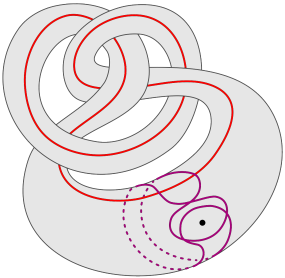

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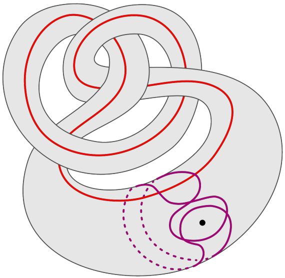

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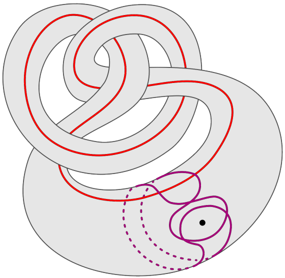

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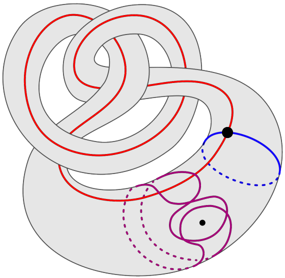

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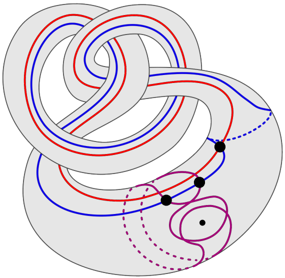

$$\Sigma(2,3,7) = S^3_{+1}(4_1) = (\text{figure-8 knot complement}) \cup (D^2 \times S^1)$$

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L-space conjecture [Boyer-Gordon-Watson]

codimension 1 structures *(taut foliations)* ↔ *fundamental group* *(left-orderability)*

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Immersed curves for 4-ended tangles I

$$\begin{pmatrix}\text{4-ended tangle } T \\ \text{in a 3-dim. ball } B^3\end{pmatrix} \xmapsto{\text{[Z'17]}} \begin{pmatrix}\text{immersed curves* } \mathrm{HFT}(T) \\ \text{on } \partial B^3 \smallsetminus \partial T\end{pmatrix}$$

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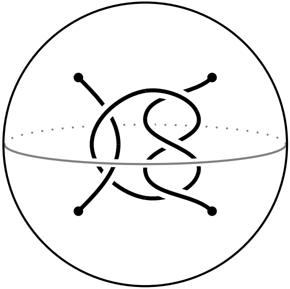

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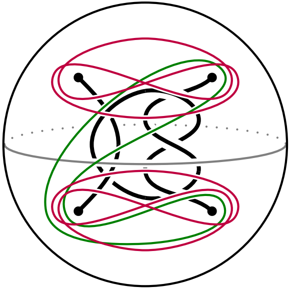

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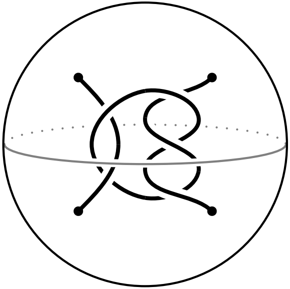

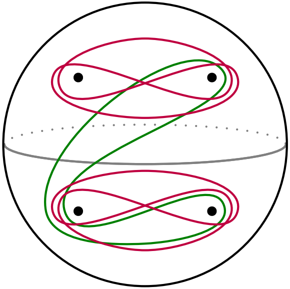

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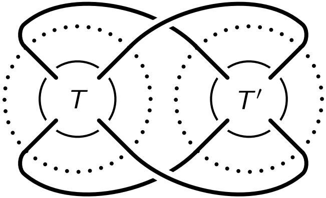

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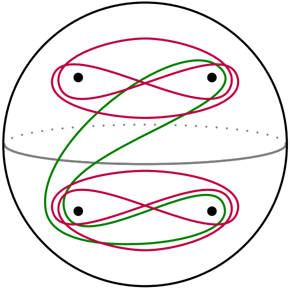

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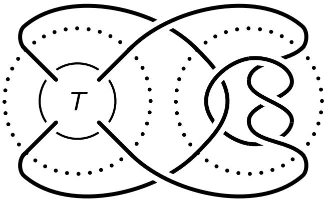

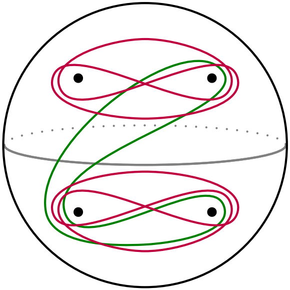

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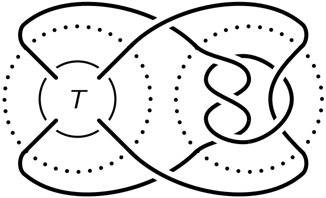
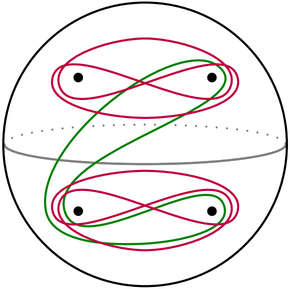


Conway mutation



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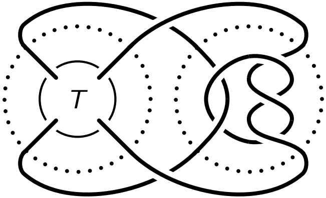

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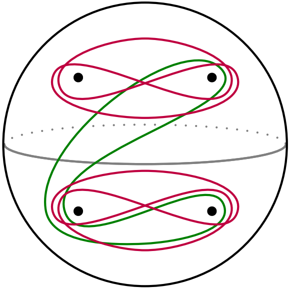

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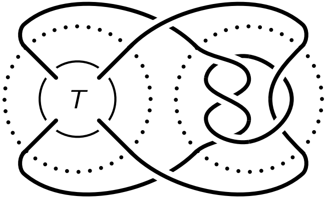

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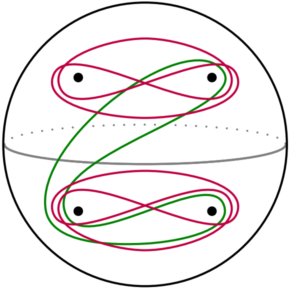

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Theorem (conjectured by Baldwin-Levine, Z'19)

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Both proofs use strong geography results for components of HFT.

Comparison of the immersed curve invariants

Template ⟨surface $\mathcal{S}$, homology theory $\mathcal{H}$⟩ invariant $\mathcal{I}$

$$\begin{pmatrix} \textit{3-dim. objects } X \\ \textit{with } \partial X = \mathcal{S} \end{pmatrix} \mapsto \begin{pmatrix} \textit{immersed curves* } \mathcal{I}(X) \\ \textit{on } \partial X = \mathcal{S} \end{pmatrix}$$

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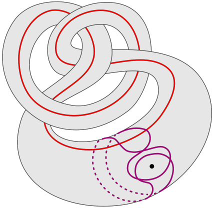
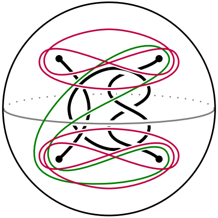
$$\mathcal{H}(X \cup_{\mathcal{S}} X') = \textit{Lagrangian Floer homology of } \mathcal{I}(X) \textit{ and } \mathcal{I}(X')$$



$\mathcal{I} = \widehat{\mathrm{HF}}$
$\mathcal{S} = T^2 \setminus (1 \text{ point})$
$\mathcal{H} =$ Heegaard Floer homology of closed 3-dim. manifolds



$\mathcal{I} = \mathrm{HFT}$
$\mathcal{S} = S^2 \setminus (4 \text{ points})$
$\mathcal{H} =$ Heegaard Floer homology of links in S^3

Immersed curves for 4-ended tangles II

[Kotelskiy-Watson-Z'19]

$$\left(\begin{array}{c}\text{pointed 4-ended tangle}\\ T \text{ in a 3-dim. ball } B^3\end{array}\right) \longmapsto \left(\begin{array}{c}\text{immersed curves* } \widetilde{\mathrm{BN}}(T)\\ \text{on } \partial B^3 \smallsetminus \partial T\end{array}\right)$$

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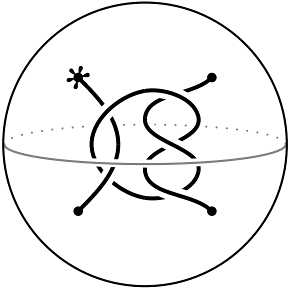

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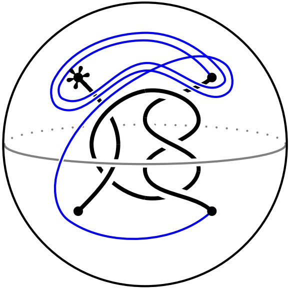

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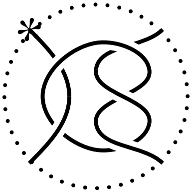

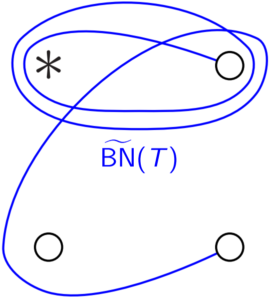

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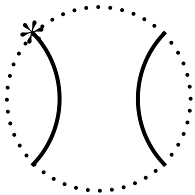

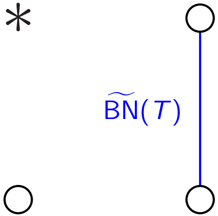

Immersed curves for 4-ended tangles II

[Kotelskiy-Watson-Z'19]

$$\left(\begin{array}{c}\text{pointed 4-ended tangle}\\ T \text{ in a 3-dim. ball } B^3\end{array}\right) \longmapsto \left(\begin{array}{c}\text{immersed curves* } \widetilde{\mathrm{BN}}(T)\\ \text{on } \partial B^3 \smallsetminus \partial T\end{array}\right)$$

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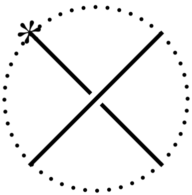

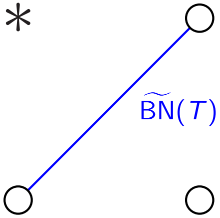

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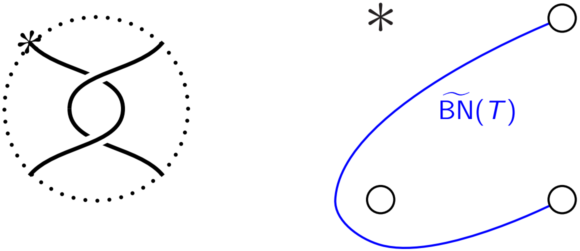

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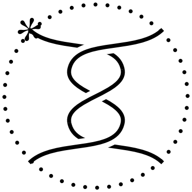

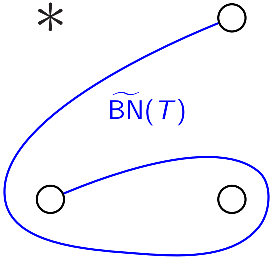

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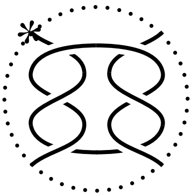

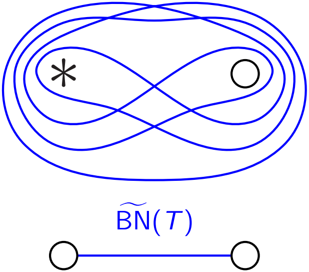

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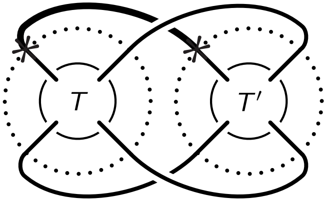
gluing theorem:

*) plus local systems $X \in \mathrm{GL}_n(\mathbf{k})$

$\widetilde{\mathrm{BN}}(T \cup T') =$ wrapped Lagrangian Floer homology of $\widetilde{\mathrm{BN}}(T)$ and $\widetilde{\mathrm{BN}}(T')$



$T \cup T'$

Immersed curves for 4-ended tangles II

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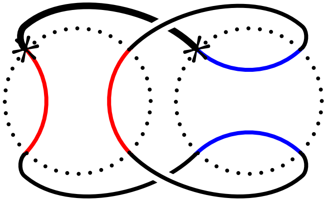
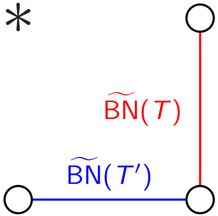


$\widetilde{\mathrm{BN}}(\text{unknot}) = \mathbf{k}[H]$



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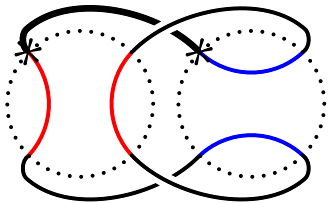

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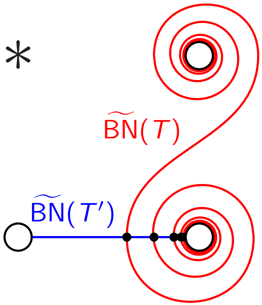

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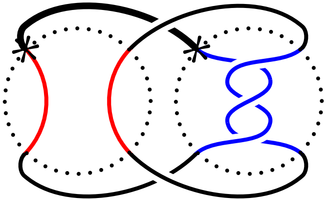
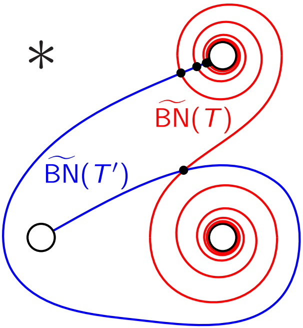
gluing theorem:

$\widetilde{\mathrm{BN}}(T \cup T') =$ wrapped Lagrangian Floer homology of $\widetilde{\mathrm{BN}}(T)$ and $\widetilde{\mathrm{BN}}(T')$



$\widetilde{\mathrm{BN}}(\text{trefoil}) = \mathbf{k} \oplus \mathbf{k}[H]$



Immersed curves for 4-ended tangles II

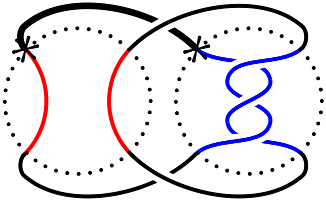
[Kotelskiy-Watson-Z'19]

$$\begin{pmatrix} \text{pointed 4-ended tangle} \\ T \text{ in a 3-dim. ball } B^3 \end{pmatrix} \longmapsto \begin{pmatrix} \text{immersed curves* } \widetilde{\mathrm{Kh}}(T) \\ \text{on } \partial B^3 \smallsetminus \partial T \end{pmatrix}$$

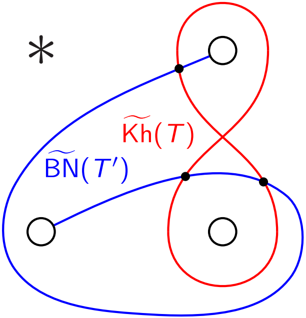
gluing theorem:

*) plus local systems $X \in \mathrm{GL}_n(\mathbf{k})$

$\widetilde{\mathrm{Kh}}(T \cup T') =$ wrapped Lagrangian Floer homology of $\widetilde{\mathrm{Kh}}(T)$ and $\widetilde{\mathrm{BN}}(T')$



$\widetilde{\mathrm{Kh}}(\text{trefoil}) = \mathbf{k}^3$



Immersed curves for 4-ended tangles II

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gluing theorem:

$\widetilde{\mathrm{BN}}(T \cup T') =$ wrapped Lagrangian Floer homology of $\widetilde{\mathrm{BN}}(T)$ and $\widetilde{\mathrm{BN}}(T')$

Theorem (Kotelskiy-Watson-Z'19)

Bar-Natan homology over $\mathbb{F}_2$ is preserved under Conway mutation. Moreover, Rasmussen's s-invariant (over any field) is preserved under Conway mutation for any knot K.

Immersed curves for 4-ended tangles II

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gluing theorem:

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Theorem (Kotelskiy-Watson-Z'19)

Bar-Natan homology over $\mathbb{F}_2$ is preserved under Conway mutation. Moreover, Rasmussen's s-invariant (over any field) is preserved under Conway mutation for any knot K.

Conjecture

Components of $\widetilde{\mathrm{Kh}}$ satisfy geography restrictions similar to HFT.

Comparison of the immersed curve invariants

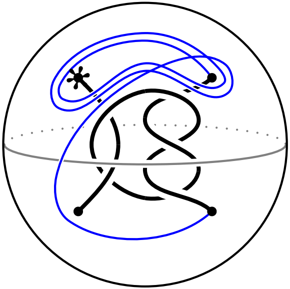
Template $\langle$surface $\mathcal{S}$, homology theory $\mathcal{H}\rangle$ invariant $\mathcal{I}$

$$\begin{pmatrix} \textit{3-dim. objects } X \\ \textit{with } \partial X = \mathcal{S} \end{pmatrix} \mapsto \begin{pmatrix} \textit{immersed curves* } \mathcal{I}(X) \\ \textit{on } \partial X = \mathcal{S} \end{pmatrix}$$

gluing theorem:

**) plus local systems $X \in \mathrm{GL}_n(\mathbf{k})$*

$\mathcal{H}(X \cup_{\mathcal{S}} X')$ = *wrapped Lagrangian Floer homology of* $\mathcal{I}(X)$ *and* $\mathcal{I}(X')$



$\mathcal{I} = \widetilde{\mathrm{BN}}$
$\mathcal{S} = S^2 \smallsetminus (4 \text{ points})$
$\mathcal{H}$ = Bar-Natan homology of links in S^3

Origins of $\widetilde{\mathrm{BN}}$ and $\widetilde{\mathrm{Kh}}$

for knots and links
Khovanov and Bar-Natan homology [Khovanov, Bar-Natan]

$\rightsquigarrow$

for tangles
Cobordism category framework [Bar-Natan]

$\rightsquigarrow$

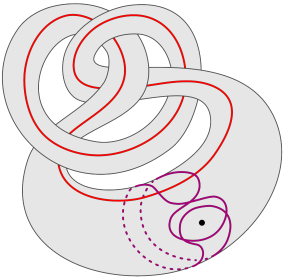
immersed curve invariants $\widetilde{\mathrm{BN}}$ **and** $\widetilde{\mathrm{Kh}}$

||

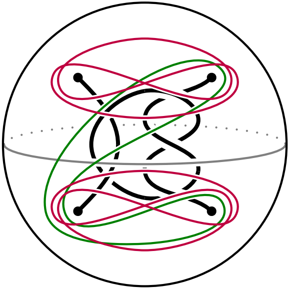
geometric reformulation of Bar-Natan's tangle theory
in the special case of 4-ended tangles

Summary

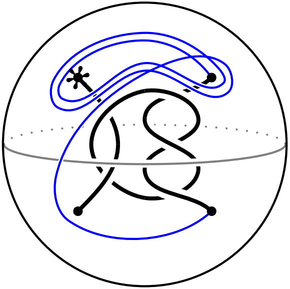
$\widehat{\mathrm{HF}}(M)$



$\mathrm{HFT}(T)$



$\widetilde{\mathrm{BN}}(T)$



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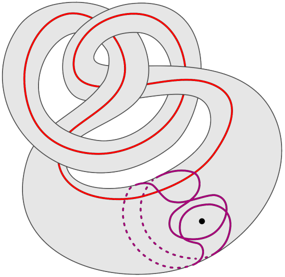

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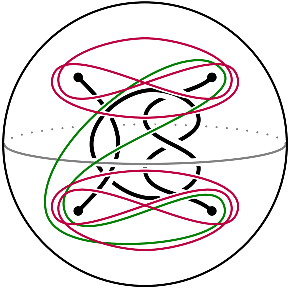

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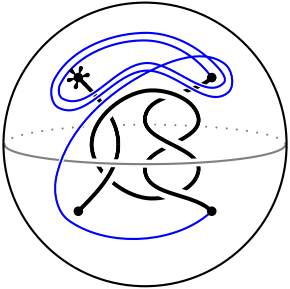

Thank you!