Conway mutation in link Floer, Khovanov and Bar-Natan homology

Claudius Zibrowius

University of British Columbia

Overview
- Conway mutation
- Conway mutation and HFL
- Conway mutation and Kh

Heegaard Floer homology for 4-ended tangles
- An immersed curve invariant
- Glueing theorem
- Proof of mutation invariance

Khovanov homology for 4-ended tangles
- Bar-Natan's invariant for 4-ended tangles
- A classification theorem
- An immersed curve invariant
- Reduced Bar-Natan homology
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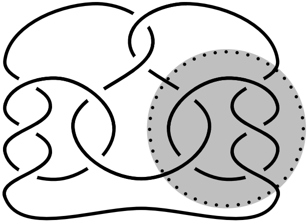
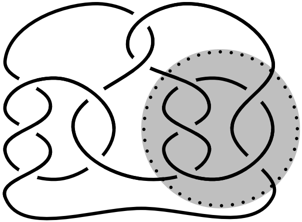
- Mutation results
- Proof of mutation invariance

Summary

Conway mutation



Kinoshita-Terasaka knot



Conway knot

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
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Reduced Bar-Natan homology
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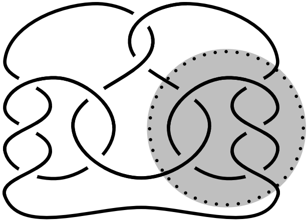

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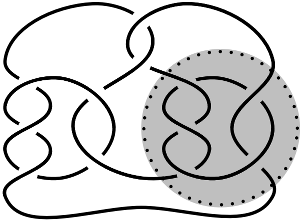

Conway knot

$$\text{Alexander polynomial } \Delta_L(t) = \chi(\widehat{\mathrm{HFL}}(L; a, h)) = \sum (-1)^h \operatorname{rk}(\widehat{\mathrm{HFL}}(L; a, h)) \cdot t^a$$

categorification ↓

$$\text{link Floer homology } \widehat{\mathrm{HFL}}(L) = \bigoplus_{(a,h)\in\mathbb{Z}^2} \widehat{\mathrm{HFL}}(L; a, h)$$

[Ozsváth-Szabó, Rasmussen '02]

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
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Proof of mutation invariance
Khovanov homology for 4-ended tangles
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An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
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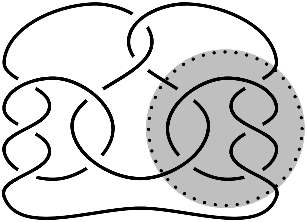

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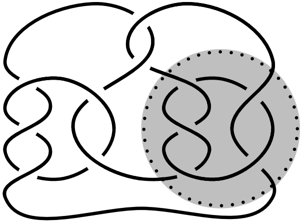

Conway knot

Jones polynomial $V(q) = \chi(\mathsf{Kh}(L; a, h))$

$$= \sum (-1)^h \, \mathsf{rk}(\mathsf{Kh}(L; a, h)) \cdot q^a$$

categorification ↓

Khovanov homology $\mathsf{Kh}(L) = \bigoplus_{(a,h) \in \mathbb{Z}^2} \mathsf{Kh}(L; a, h)$

[Khovanov '99]

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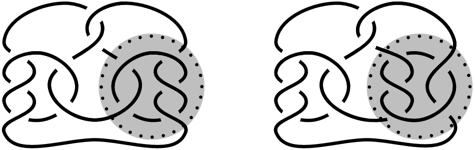
Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Conway mutation and $\widehat{\text{HFL}}$



Example ([Ozváth-Szabó '03])

$\widehat{\text{HFL}}$(Kinoshita-Terasaka knot) $\neq$ $\widehat{\text{HFL}}$(Conway knot) as bigraded Abelian groups.

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
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Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

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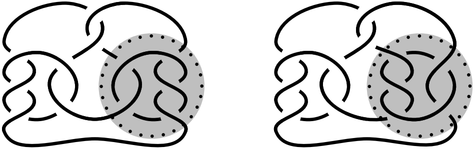

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Conjecture ([Baldwin-Levine '11])

Conway mutation preserves δ-graded $\widehat{\mathrm{HFL}}$, defined by

$$\widehat{\mathrm{HFL}}_\delta(L) := \bigoplus_{a-h=\delta} \widehat{\mathrm{HFL}}(L; a, h) \quad \textit{for } \delta \in \mathbb{Z}$$

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

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Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
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Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

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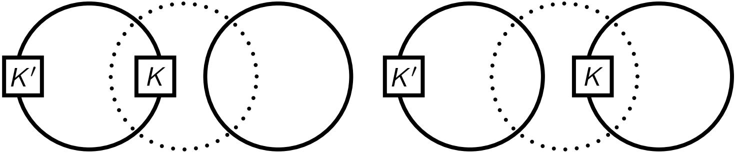

Example ([Wehrli '03])

$\dim_{\mathbb{Q}} \mathrm{Kh}(K \# K' \cup \bigcirc; \mathbb{Q}) \neq \dim_{\mathbb{Q}} \mathrm{Kh}(K \cup K'; \mathbb{Q})$

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
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Proof of mutation invariance
Summary

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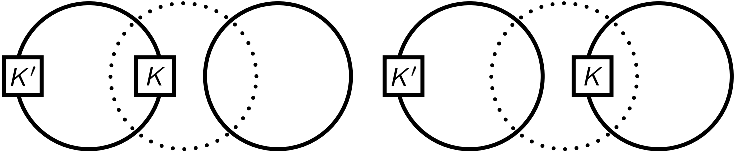

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Theorem ([Wehrli, Bloom '09])

Conway mutation preserves $\mathrm{Kh}(L; \mathbb{F}_2)$*.*

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
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An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

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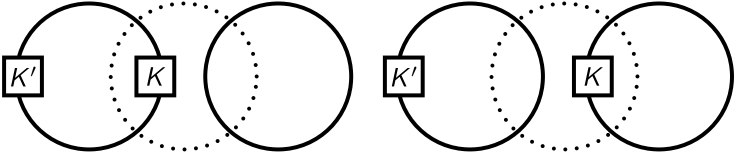

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Conjecture

Conway mutation preserves $\mathrm{Kh}(K)$ *of knots* K.

Overview
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Conway mutation and HFL
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Heegaard Floer homology for 4-ended tangles
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Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
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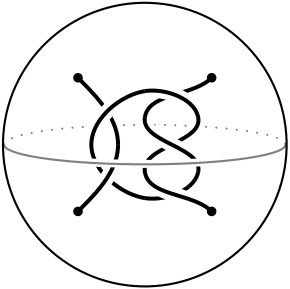
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

The immersed curve invariant HFT [Z '17]

$$\frac{\left\{\begin{array}{c}\text{4-ended}\\ \text{tangles } T\subset D^3\end{array}\right\}}{\text{isotopy}} \longrightarrow \frac{\left\{\begin{array}{c}\text{immersed curves* on}\\ S^2 \smallsetminus 4 = \partial D^3 \smallsetminus \partial T\end{array}\right\}}{\text{homotopy}}$$

$$T \longmapsto \mathrm{HFT}(T)$$



*) plus local systems $X \in \mathrm{GL}_n(\mathbb{F}_2)$

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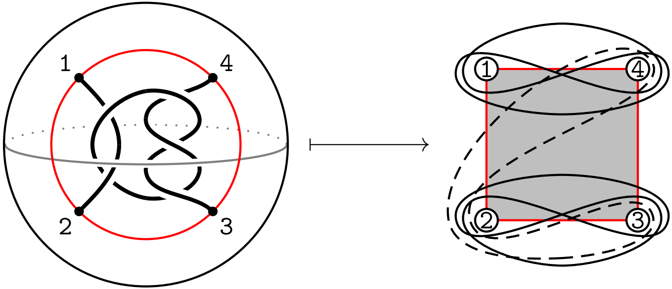

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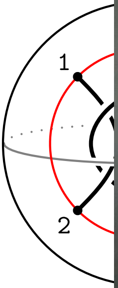

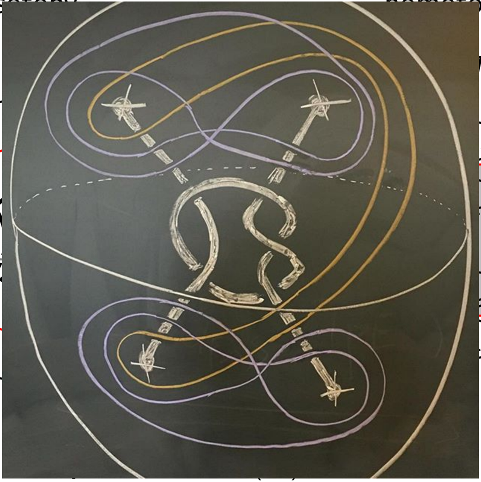

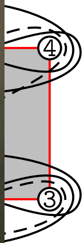

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Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh

Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance

Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance

Summary

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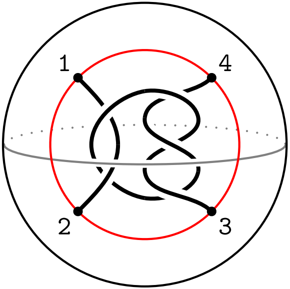

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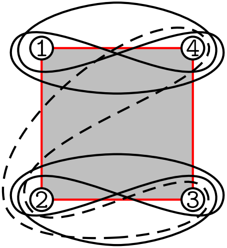

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Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

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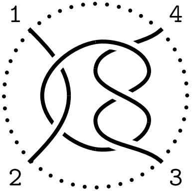

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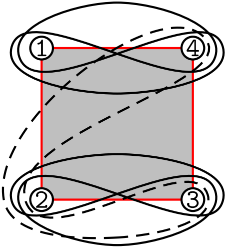

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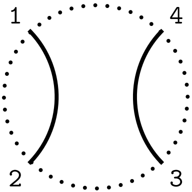

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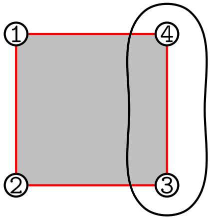

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Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

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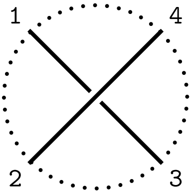

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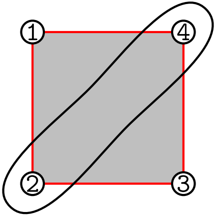

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Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

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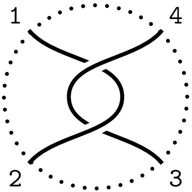

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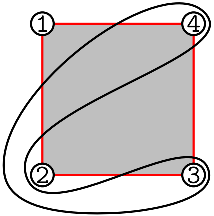

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Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

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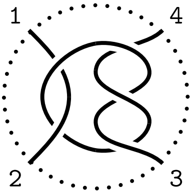

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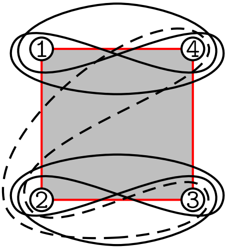

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Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

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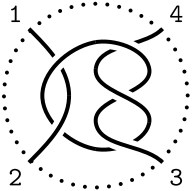

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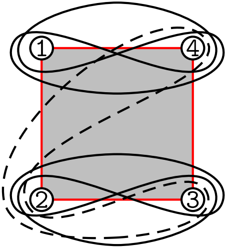

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Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
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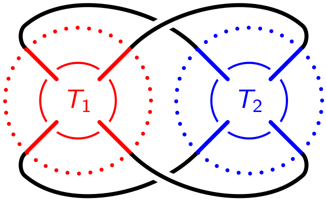
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Glueing theorem for HFT

Theorem ([Z '17])

$\widehat{\mathrm{HFL}}(L) \otimes \mathbb{F}_2^i \cong \mathrm{HF}(\mathrm{mr}(\mathrm{HFT}(T_1)), \mathrm{HFT}(T_2))$ *where*

- HF *is Lagrangian Floer homology*
- $\mathrm{mr}\colon \partial D^3 \smallsetminus \partial T_1 \longrightarrow \partial D^3 \smallsetminus \partial T_2$
- $i \in \{1,2\}$ *(depending on #components in L)*

$L :=$ 

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
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Reduced Bar-Natan homology
Glueing theorem
Mutation results
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Summary

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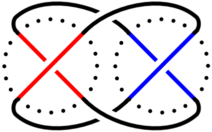
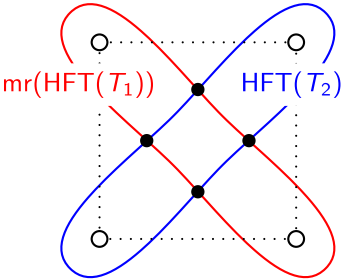


Hopf link



Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Glueing theorem for HFT

Theorem ([Z '17])

$\widehat{\mathrm{HFL}}(L) \otimes \mathbb{F}_2^i \cong \mathrm{HF}(\mathrm{mr}(\mathrm{HFT}(T_1)), \mathrm{HFT}(T_2))$ *where*

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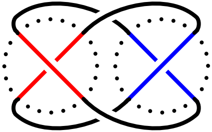

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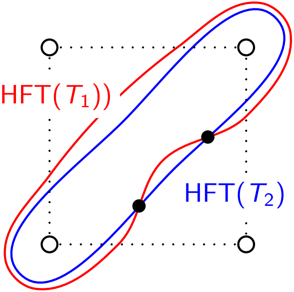

Conway mutation in link Floer, Khovanov and Bar-Natan homology

Claudius Zibrowius

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Glueing theorem for HFT

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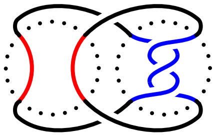
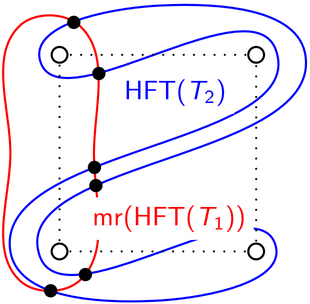


trefoil knot



Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

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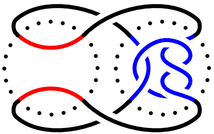
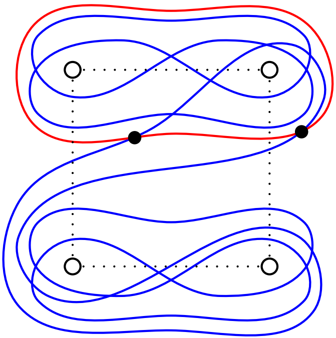


unknot



Overview
- Conway mutation
- Conway mutation and HFL
- Conway mutation and Kh

Heegaard Floer homology for 4-ended tangles
- An immersed curve invariant
- Glueing theorem
- Proof of mutation invariance

Khovanov homology for 4-ended tangles
- Bar-Natan's invariant for 4-ended tangles
- A classification theorem
- An immersed curve invariant
- Reduced Bar-Natan homology
- Glueing theorem
- Mutation results
- Proof of mutation invariance

Summary

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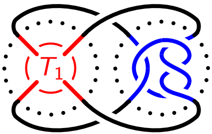

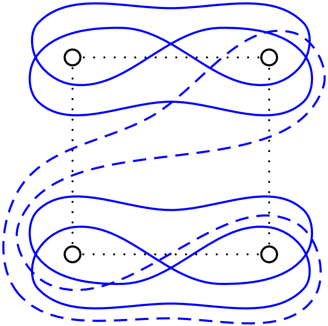

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

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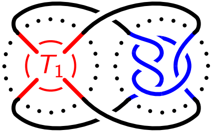
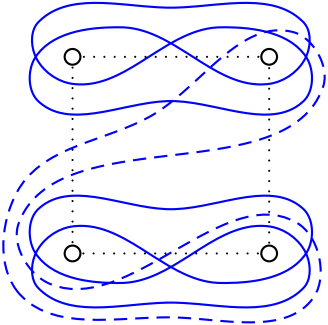




Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Proof of mutation invariance

Theorem ([Z '19])

Conway mutation preserves relatively δ-graded $\widehat{\mathrm{HFL}}(L;\mathbb{F}_2)$.

Overview

Conway mutation

Conway mutation and HFL

Conway mutation and Kh

Heegaard Floer homology for 4-ended tangles

An immersed curve invariant

Glueing theorem

Proof of mutation invariance

Khovanov homology for 4-ended tangles

Bar-Natan's invariant for 4-ended tangles

A classification theorem

An immersed curve invariant

Reduced Bar-Natan homology

Glueing theorem

Mutation results

Proof of mutation invariance

Summary

Proof of mutation invariance

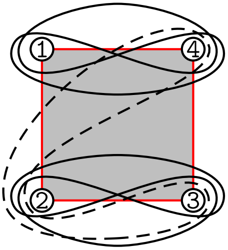
Theorem ([Z '19])

Conway mutation preserves relatively δ-graded $\widehat{\mathrm{HFL}}(L;\mathbb{F}_2)$.

Sketch proof.

- Conjugation symmetry:

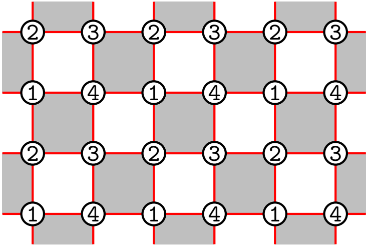
$$\mathrm{Conj}(\mathrm{HFT}(T)) = \mathrm{HFT}(T) \otimes \mathbb{F}_2^4$$



Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

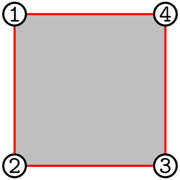
Proof of mutation invariance

- Linearity of components of HFT(T):



$\mathbb{R}^2 \smallsetminus \mathbb{Z}^2$

$\longrightarrow$

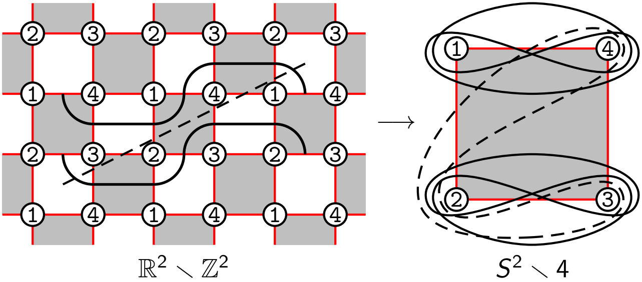


$S^2 \smallsetminus 4$

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

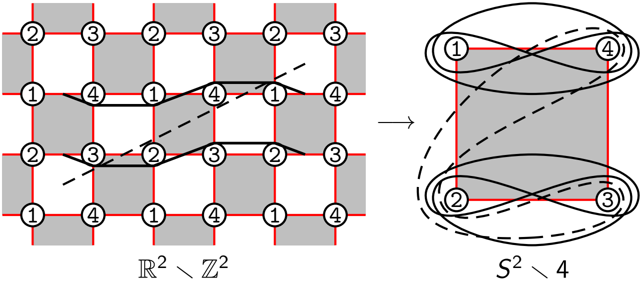
Proof of mutation invariance

- Linearity of components of $\mathrm{HFT}(T)$:



Proof of mutation invariance

- Linearity of components of HFT(T):



Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Proof of mutation invariance

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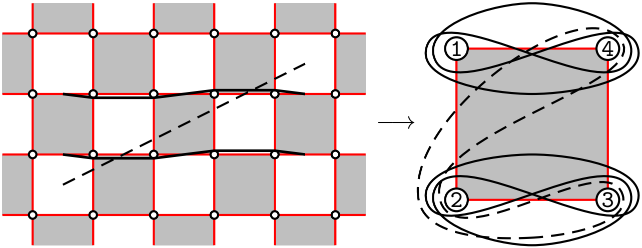

$\mathbb{R}^2 \smallsetminus \mathbb{Z}^2$ $\qquad$ $S^2 \smallsetminus 4$

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Proof of mutation invariance

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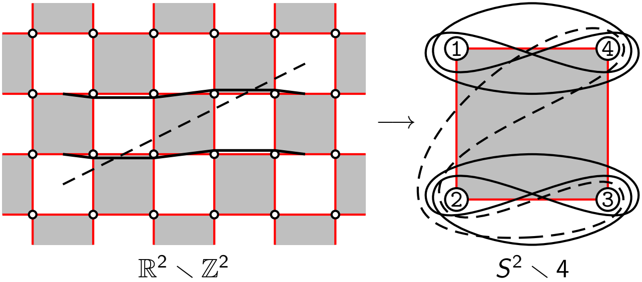


Lemma

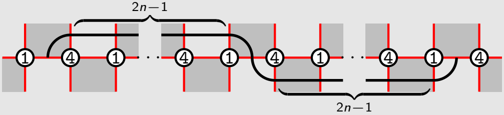
All components of HFT(T) *lift to linear curves.*

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

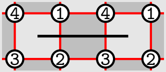
Proof of mutation invariance

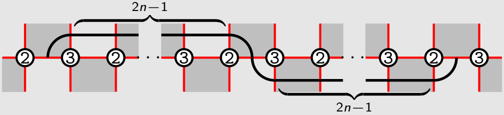
Proposition

Up to twisting, each component of $\mathrm{HFT}(T)$ *lifts to*

$\mathfrak{d}_n :=$ 

, $n \in \mathbb{N}$

$\mathfrak{r}_X :=$  *for some local system* $X \in \mathrm{GL}_n(\mathbb{F}_2)$, *or*

$\mathfrak{b}_n :=$ 

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Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Proof of mutation invariance

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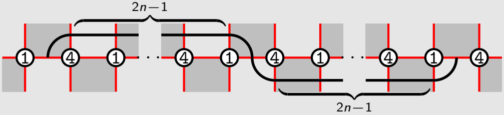
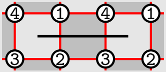
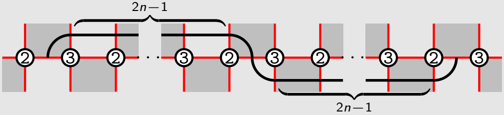
$\mathfrak{d}_n :=$ 

$, \; n \in \mathbb{N}$

$\mathfrak{r}_X :=$  *for some local system* $X \in \mathrm{GL}_n(\mathbb{F}_2)$*, or*

$\mathfrak{b}_n :=$ 

$, \; n \in \mathbb{N}$

- Brute force computation: $\mathrm{Conj}(\mathfrak{r}_X) = \mathfrak{r}_{X^{-1}} \otimes \mathbb{F}_2^4$

$$\mathrm{Conj}(\mathfrak{d}_n) = \mathfrak{b}_n \otimes \mathbb{F}_2^4 \quad \text{and} \quad \mathrm{Conj}(\mathfrak{b}_n) = \mathfrak{d}_n \otimes \mathbb{F}_2^4$$

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Proof of mutation invariance

Sketch proof.

- Conjugation symmetry:

$$\mathrm{Conj}(\mathrm{HFT}(T)) = \mathrm{HFT}(T) \otimes \mathbb{F}_2^4$$

- Linearity of components of $\mathrm{HFT}(T)$:

$\mathrm{HFT}(T)$ consists of linear curves $\mathfrak{d}_n$, $\mathfrak{r}_X$, and $\mathfrak{b}_n$.

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□

Khovanov homology for 4-ended tangles

joint work with Artem Kotelskiy (IU)
and Liam Watson (UBC)

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Bar-Natan's invariant for 4-ended tangles

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

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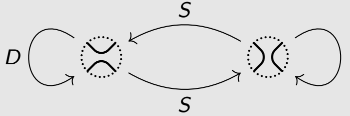
$\mathcal{B} := \Bbbk\Big[$ 

$\Big] \Big/ (DS = 0 = SD)$

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Bar-Natan's invariant for 4-ended tangles

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$$\mathcal{B} := \Bbbk\Big[D \circlearrowleft \bullet \underset{S}{\overset{S}{\rightleftarrows}} \circ \circlearrowright D \Big] \Big/ (DS = 0 = SD)$$

Bar-Natan's invariant for 4-ended tangles

Definition

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$$\dfrac{\left\{ \begin{array}{c} \text{4-ended} \\ \text{tangles } T \subset D^3 \end{array} \right\}}{\text{isotopy}} \longrightarrow \dfrac{\left\{ \begin{array}{c} \text{chain complexes} \\ \text{over the algebra } \mathcal{B} \end{array} \right\}}{\text{chain homotopy}}$$

$$T \longmapsto \text{Д}(T)$$

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Bar-Natan's invariant for 4-ended tangles

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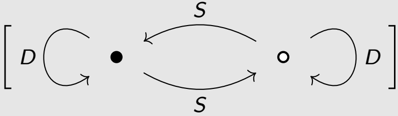
$\mathcal{B} := \Bbbk$ 

$/(DS = 0 = SD)$

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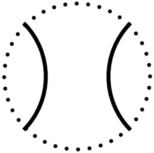 $\longmapsto [\circ]$

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

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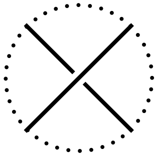

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Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

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Definition

$$\mathcal{B} := \Bbbk\left[D \circlearrowright \bullet \underset{S}{\overset{S}{\rightleftarrows}} \circ \circlearrowleft D \right] \Big/ (DS = 0 = SD)$$

$$\frac{\left\{ \begin{array}{c} \text{4-ended} \\ \text{tangles } T \subset D^3 \end{array} \right\}}{\text{isotopy}} \longrightarrow \frac{\left\{ \begin{array}{c} \text{chain complexes} \\ \text{over the algebra } \mathcal{B} \end{array} \right\}}{\text{chain homotopy}}$$

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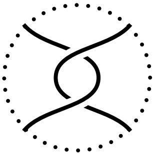


$$\longmapsto \left[\circ \xrightarrow{S} \bullet \xrightarrow{D} \bullet \right]$$

Overview
- Conway mutation
- Conway mutation and HFL
- Conway mutation and Kh

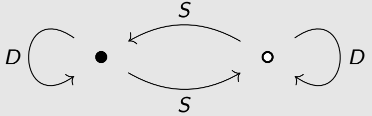
Heegaard Floer homology for 4-ended tangles
- An immersed curve invariant
- Glueing theorem
- Proof of mutation invariance

Khovanov homology for 4-ended tangles
- Bar-Natan's invariant for 4-ended tangles
- A classification theorem
- An immersed curve invariant
- Reduced Bar-Natan homology
- Glueing theorem
- Mutation results
- Proof of mutation invariance

Summary

Bar-Natan's invariant for 4-ended tangles

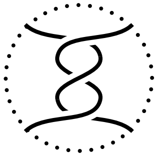
Definition

$\mathcal{B} := \Bbbk\Big[$ 

$\Big]\Big/(DS = 0 = SD)$

$$\frac{\left\{\begin{array}{c}\text{4-ended}\\ \text{tangles } T \subset D^3\end{array}\right\}}{\text{isotopy}} \longrightarrow \frac{\left\{\begin{array}{c}\text{chain complexes}\\ \text{over the algebra } \mathcal{B}\end{array}\right\}}{\text{chain homotopy}}$$

$$T \longmapsto Д(T)$$



$$\longmapsto \left[\circ \xrightarrow{S} \bullet \xrightarrow{D} \bullet \xrightarrow{S^2} \bullet\right]$$

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

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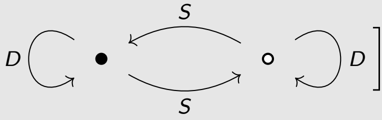
$\mathcal{B} := \mathbb{k}\Big[$  $\Big] \Big/ (DS = 0 = SD)$

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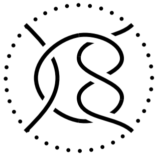
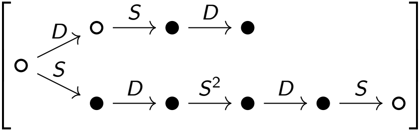
 $\longmapsto$ 

$$\left[\begin{array}{l} \circ \xrightarrow{D} \circ \xrightarrow{S} \bullet \xrightarrow{D} \bullet \\ \circ \xrightarrow{S} \bullet \xrightarrow{D} \bullet \xrightarrow{S^2} \bullet \xrightarrow{D} \bullet \xrightarrow{S} \circ \end{array}\right]$$

Overview
- Conway mutation
- Conway mutation and HFL
- Conway mutation and Kh

Heegaard Floer homology for 4-ended tangles
- An immersed curve invariant
- Glueing theorem
- Proof of mutation invariance

Khovanov homology for 4-ended tangles
- Bar-Natan's invariant for 4-ended tangles
- A classification theorem
- An immersed curve invariant
- Reduced Bar-Natan homology
- Glueing theorem
- Mutation results
- Proof of mutation invariance

Summary

Bar-Natan's invariant for 4-ended tangles

Definition

$$\mathcal{B} := \Bbbk\Big[D \circlearrowleft \bullet \underset{S}{\overset{S}{\rightleftarrows}} \circ \circlearrowright D \Big] \Big/ (DS = 0 = SD)$$

$$\frac{\left\{\begin{matrix}\textbf{pointed } \text{4-ended} \\ \text{tangles } T \subset D^3\end{matrix}\right\}}{\text{isotopy}} \longrightarrow \frac{\left\{\begin{matrix}\text{chain complexes} \\ \text{over the algebra } \mathcal{B}\end{matrix}\right\}}{\text{chain homotopy}}$$

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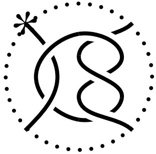

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Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

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[redacted]

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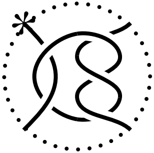

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Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Motivation for defining HFT

Can I make $\widehat{\mathrm{HFL}}$ *look more like Bar-Natan's local version of Khovanov homology for tangles?*

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Motivation for defining HFT

Can I make $\widehat{\mathrm{HFL}}$ *look more like Bar-Natan's local version of Khovanov homology for tangles?*

$\rightsquigarrow$

Question

Can we make Bar-Natan's local version of Khovanov homology for 4-ended tangles look more like HFT*?*

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

A classification theorem

$$\frac{\left\{\begin{matrix}\text{chain complexes}\\ \text{over the } \mathbb{k}\text{-algebra } \mathcal{B}\end{matrix}\right\}}{\text{chain homotopy}} \overset{1:1}{\longleftrightarrow} \frac{\left\{\begin{matrix}\text{immersed curves* on}\\ \text{a 4-punctured sphere } S^2_{4,*}\end{matrix}\right\}}{\text{homotopy}}$$

*) plus local systems $X \in \mathrm{GL}_n(\mathbb{k})$

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

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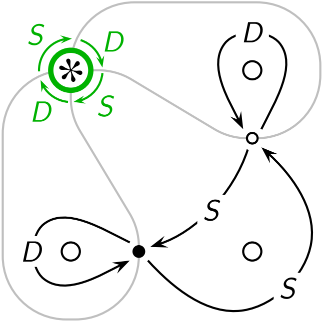

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Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

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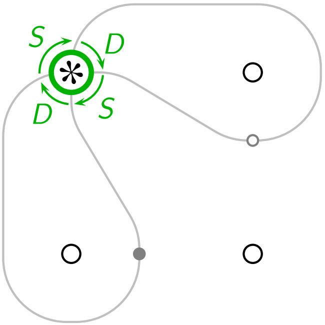

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Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

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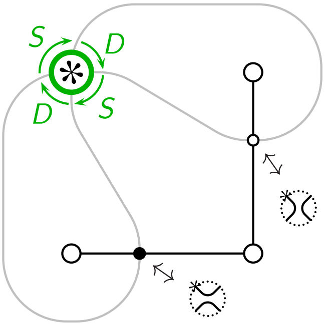

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Overview
- Conway mutation
- Conway mutation and HFL
- Conway mutation and Kh

Heegaard Floer homology for 4-ended tangles
- An immersed curve invariant
- Glueing theorem
- Proof of mutation invariance

Khovanov homology for 4-ended tangles
- Bar-Natan's invariant for 4-ended tangles
- A classification theorem
- An immersed curve invariant
- Reduced Bar-Natan homology
- Glueing theorem
- Mutation results
- Proof of mutation invariance

Summary

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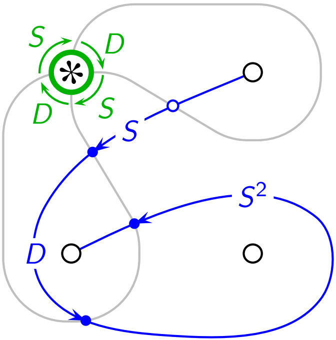

$$\left[\circ \xrightarrow{S} \bullet \xrightarrow{D} \bullet \xrightarrow{S^2} \bullet\right]$$

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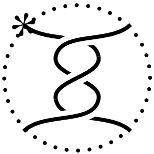
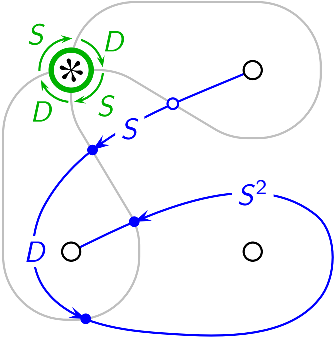
Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

An immersed curve invariant [KWZ'19]

$$\frac{\left\{\begin{array}{c}\textbf{pointed } \text{4-ended} \\ \text{tangles } T \subset D^3\end{array}\right\}}{\text{isotopy}} \longrightarrow \frac{\left\{\begin{array}{c}\text{immersed curves* on} \\ \text{a 4-punctured sphere } S^2_{4,*}\end{array}\right\}}{\text{homotopy}}$$



$\longmapsto$



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Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

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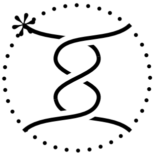

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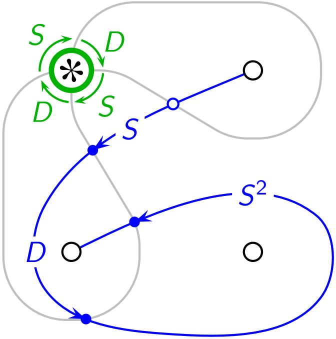

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Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

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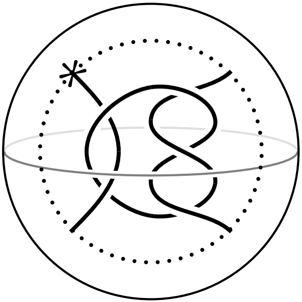

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Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

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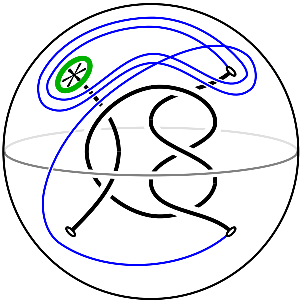

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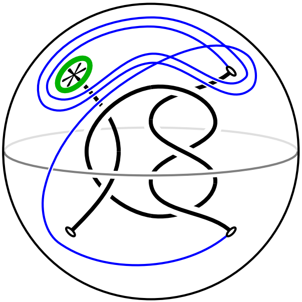
$$T \longmapsto \widetilde{\mathrm{BN}}(T; \Bbbk)$$



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Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Reduced Bar-Natan homology $\widetilde{\mathrm{BN}}(L)$

- Given a link L, $\widetilde{\mathrm{BN}}(L)$ is a bigraded $\Bbbk[H]$-module.
- There is an exact triangle of $\Bbbk$-vector spaces

$$\begin{array}{ccc} \widetilde{\mathrm{BN}}(L) & \xrightarrow{\quad \cdot H \quad} & \widetilde{\mathrm{BN}}(L) \\ & \nwarrow \qquad \swarrow & \\ & \widetilde{\mathrm{Kh}}(L) & \end{array}$$

  where $\widetilde{\mathrm{Kh}}(L)$ is the reduced Khovanov homology.
- $\widetilde{\mathrm{BN}}(K) = \Bbbk[H] \oplus (H\text{-torsion})$ for knots K;
  the quantum grading of the generator of $\Bbbk[H]$ is Rasmussen's s-invariant $s(K)$.

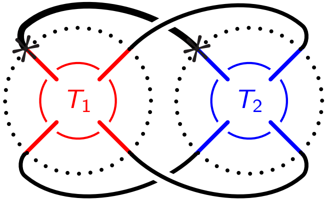
Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Glueing theorem for $\widetilde{\mathrm{BN}}$

Theorem ([KWZ'19])

$\widetilde{\mathrm{BN}}(L) \cong \mathrm{HF}(\mathrm{mr}(\widetilde{\mathrm{BN}}(T_1)), \widetilde{\mathrm{BN}}(T_2))$ *where*

- HF *is the wrapped Lagrangian Floer homology*
- $\mathrm{mr}\colon \partial D^3 \smallsetminus \partial T_1 \longrightarrow \partial D^3 \smallsetminus \partial T_2$

$L :=$ 

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

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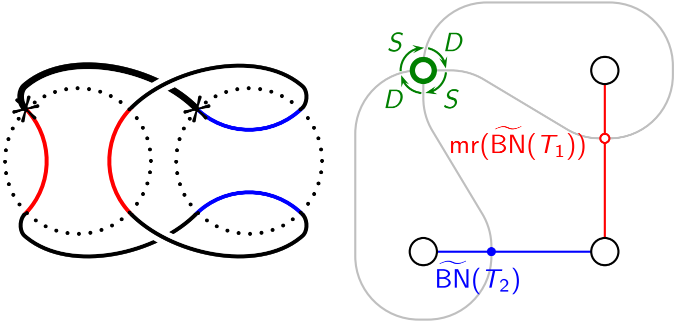

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

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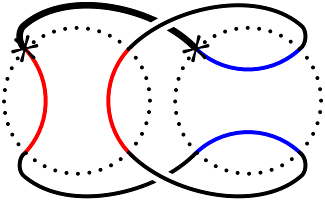
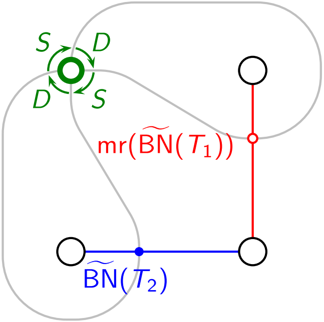


$\widetilde{\mathrm{BN}}(L) = \Bbbk[H]$



Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Glueing theorem for $\widetilde{\mathrm{BN}}$

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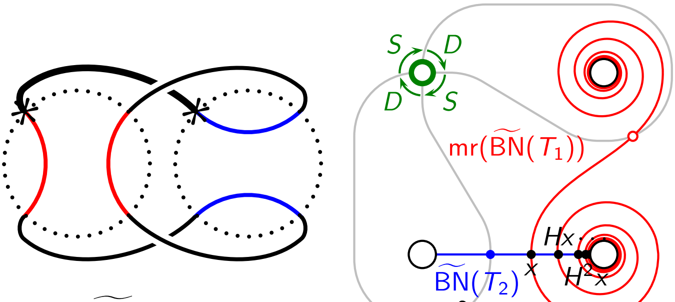


$\widetilde{\mathrm{BN}}(L) = \Bbbk[H] = \langle x, Hx, Hx^2, \dots \rangle$

Conway mutation in link Floer, Khovanov and Bar-Natan homology

Claudius Zibrowius

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh

Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance

Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance

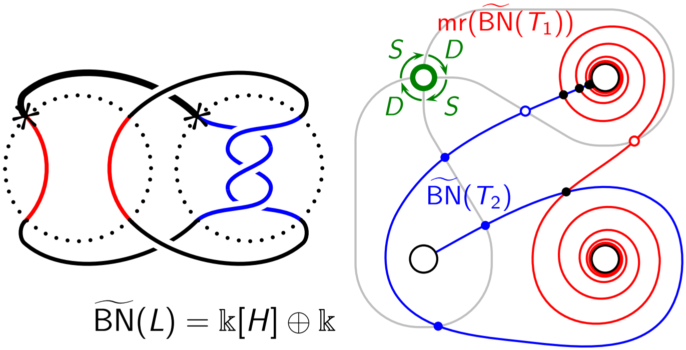
Summary

Glueing theorem for $\widetilde{\mathrm{BN}}$

Theorem ([KWZ'19])

$\widetilde{\mathrm{BN}}(L) \cong \mathrm{HF}(\mathrm{mr}(\widetilde{\mathrm{BN}}(T_1)), \widetilde{\mathrm{BN}}(T_2))$ *where*

- HF *is the wrapped Lagrangian Floer homology*
- $\mathrm{mr}\colon \partial D^3 \smallsetminus \partial T_1 \longrightarrow \partial D^3 \smallsetminus \partial T_2$



$\widetilde{\mathrm{BN}}(L) = \Bbbk[H] \oplus \Bbbk$

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Mutation results

Theorem ([KWZ '19])

*Conway mutation preserves the underlying immersed curves of $\widetilde{\mathrm{BN}}(T)$, as well as their local systems **up to multiplication by** ± 1.*

Overview
- Conway mutation
- Conway mutation and HFL
- Conway mutation and Kh

Heegaard Floer homology for 4-ended tangles
- An immersed curve invariant
- Glueing theorem
- Proof of mutation invariance

Khovanov homology for 4-ended tangles
- Bar-Natan's invariant for 4-ended tangles
- A classification theorem
- An immersed curve invariant
- Reduced Bar-Natan homology
- Glueing theorem
- Mutation results
- Proof of mutation invariance

Summary

Mutation results

Theorem ([KWZ '19])

Conway mutation preserves the underlying immersed curves of $\widetilde{\mathrm{BN}}(T)$*, as well as their local systems* ***up to multiplication by*** ± 1*.*

Corollary

Conway mutation preserves $\widetilde{\mathrm{BN}}(L; \mathbb{F}_2)$*.*

Overview

Conway mutation

Conway mutation and HFL

Conway mutation and Kh

Heegaard Floer homology for 4-ended tangles

An immersed curve invariant

Glueing theorem

Proof of mutation invariance

Khovanov homology for 4-ended tangles

Bar-Natan's invariant for 4-ended tangles

A classification theorem

An immersed curve invariant

Reduced Bar-Natan homology

Glueing theorem

Mutation results

Proof of mutation invariance

Summary

Mutation results

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Corollary

Conway mutation preserves $\widetilde{\mathrm{BN}}(L;\mathbb{F}_2)$.

Corollary

Conway mutation preserves Rasmussen's s-invariant over any field $\Bbbk$.

Overview

Conway mutation

Conway mutation and HFL

Conway mutation and Kh

Heegaard Floer homology for 4-ended tangles

An immersed curve invariant

Glueing theorem

Proof of mutation invariance

Khovanov homology for 4-ended tangles

Bar-Natan's invariant for 4-ended tangles

A classification theorem

An immersed curve invariant

Reduced Bar-Natan homology

Glueing theorem

Mutation results

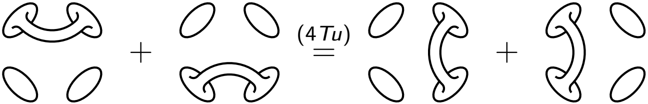
Proof of mutation invariance

Summary

Proof of mutation invariance

Relations in Bar-Natan's cobordism category $\text{Cob}_{/l}$:

$\overset{(S)}{=} 0$



Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Proof of mutation invariance

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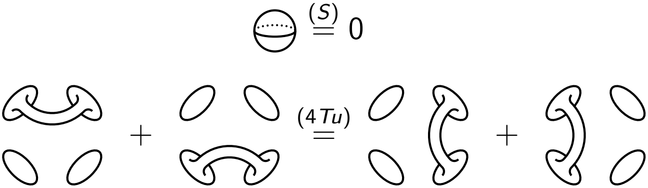
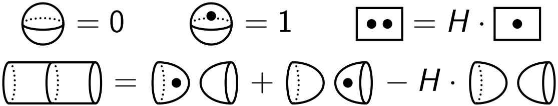


Relations in Bar-Natan's cobordism category $\mathrm{Cob}_{\bullet/l}$:



Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Proof of mutation invariance

The choice of tangle end $*$ distinguishes one component of each cobordism.

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Proof of mutation invariance

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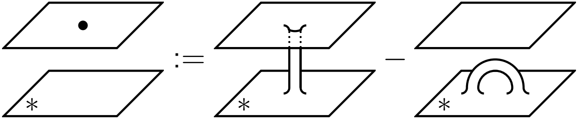

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Proof of mutation invariance

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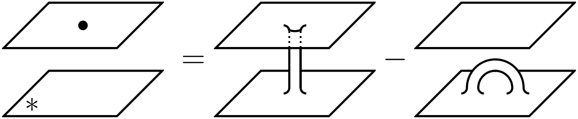

Overview

Conway mutation

Conway mutation and HFL

Conway mutation and Kh

Heegaard Floer homology for 4-ended tangles

An immersed curve invariant

Glueing theorem

Proof of mutation invariance

Khovanov homology for 4-ended tangles

Bar-Natan's invariant for 4-ended tangles

A classification theorem

An immersed curve invariant

Reduced Bar-Natan homology

Glueing theorem

Mutation results

Proof of mutation invariance

Summary

Proof of mutation invariance

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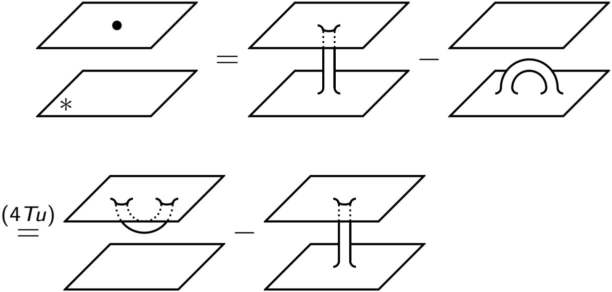

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Proof of mutation invariance

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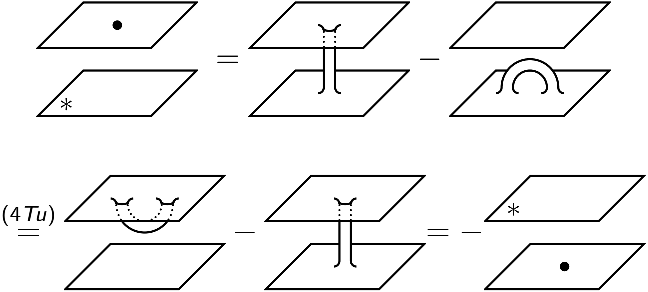
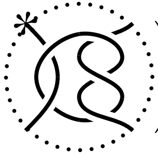
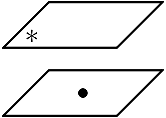


Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Proof of mutation invariance

Example:

Д () $= \left[\begin{array}{lllll} & \overset{D}{\nearrow} \circ \xrightarrow{S} \bullet \xrightarrow{D} \bullet & & & \\ \circ & & & & \\ & \underset{S}{\searrow} \bullet \xrightarrow{D} \bullet \xrightarrow{S^2} \bullet \xrightarrow{D} \bullet \xrightarrow{S} \circ & & & \end{array}\right]$

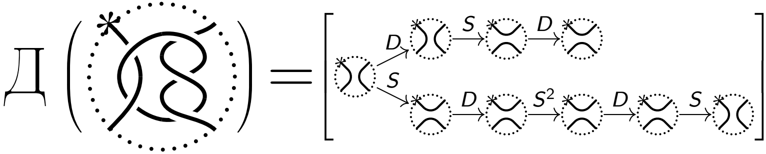
$D =$ 

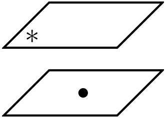
$S =$ saddle

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Proof of mutation invariance

Example:

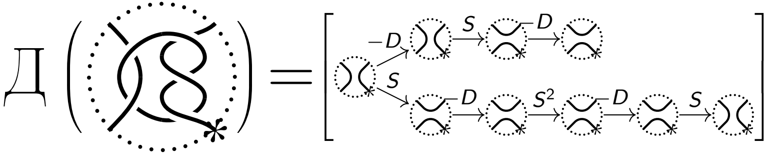


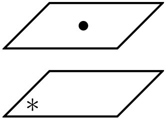
$D =$  $S =$ saddle

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Proof of mutation invariance

Example:

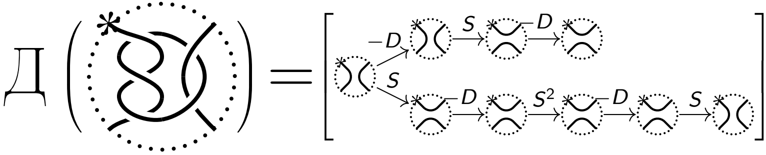


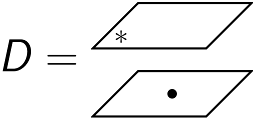
$D =$  $S =$ saddle

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Proof of mutation invariance

Example:





$S =$ saddle

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Proof of mutation invariance

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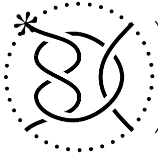
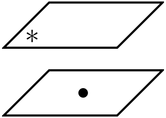
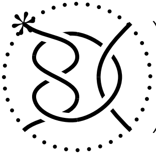
Д () $= \left[\begin{array}{l} \circ \begin{array}{l} \overset{-D}{\nearrow} \circ \overset{S}{\longrightarrow} \bullet \overset{-D}{\longrightarrow} \bullet \\ \underset{S}{\searrow} \bullet \overset{-D}{\longrightarrow} \bullet \overset{S^2}{\longrightarrow} \bullet \overset{-D}{\longrightarrow} \bullet \overset{S}{\longrightarrow} \circ \end{array} \end{array} \right]$

$D =$  $S =$ saddle

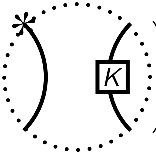
Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

Proof of mutation invariance

Example:

Д () =

$$\left[\begin{array}{l} \circ \;\begin{array}{l} \nearrow^{-D} \;\circ \xrightarrow{S} \bullet \xrightarrow{-D} \bullet \\ \searrow_{S} \;\bullet \xrightarrow{-D} \bullet \xrightarrow{S^2} \bullet \xrightarrow{-D} \bullet \xrightarrow{S} \circ \end{array} \end{array}\right]$$

Д () =

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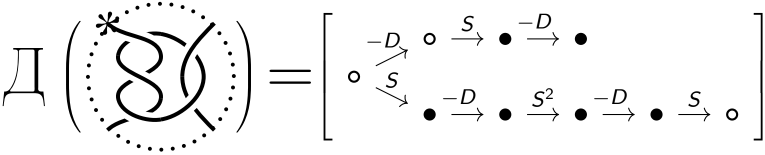
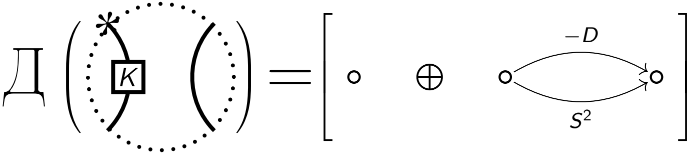
Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

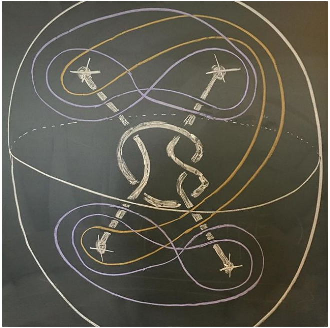
Proof of mutation invariance

Example:





Summary



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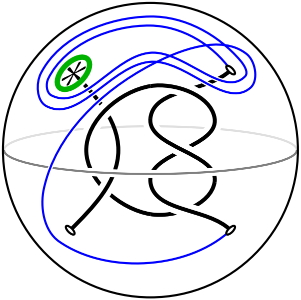
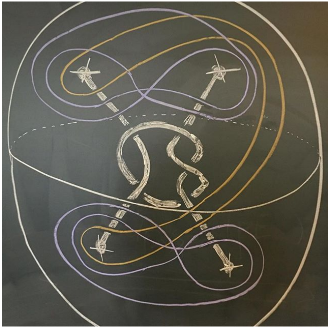
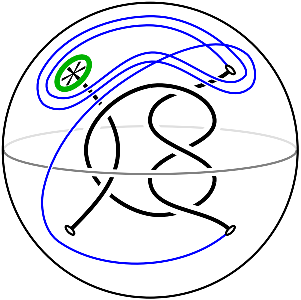


$\widetilde{\mathrm{BN}}(T)$

Overview
Conway mutation
Conway mutation and HFL
Conway mutation and Kh
Heegaard Floer homology for 4-ended tangles
An immersed curve invariant
Glueing theorem
Proof of mutation invariance
Khovanov homology for 4-ended tangles
Bar-Natan's invariant for 4-ended tangles
A classification theorem
An immersed curve invariant
Reduced Bar-Natan homology
Glueing theorem
Mutation results
Proof of mutation invariance
Summary

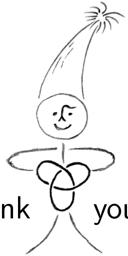
Summary





HFT(T)

$\widetilde{\mathrm{BN}}(T)$

Thank  you!

Overview

- Conway mutation
- Conway mutation and HFL
- Conway mutation and Kh

Heegaard Floer homology for 4-ended tangles

- An immersed curve invariant
- Glueing theorem
- Proof of mutation invariance

Khovanov homology for 4-ended tangles

- Bar-Natan's invariant for 4-ended tangles
- A classification theorem
- An immersed curve invariant
- Reduced Bar-Natan homology
- Glueing theorem
- Mutation results
- Proof of mutation invariance

Summary