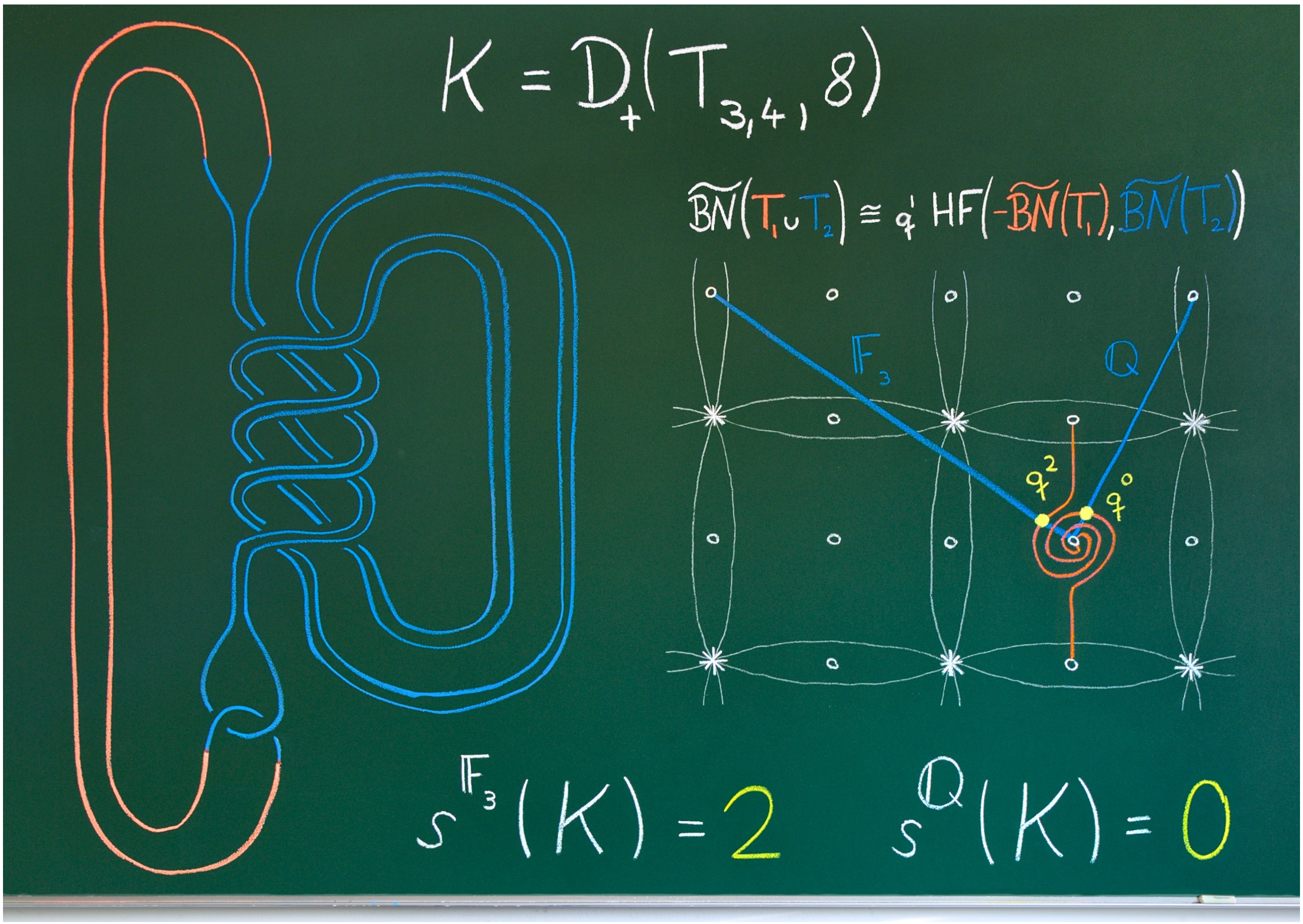

Rasmussen invariants

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June 25, 2021. In 2004, Rasmussen used Khovanov homology to define a knot invariant s, which was found to have surprisingly strong geometric applications [6]. Namely, $s/2$ is a homomorphism from the smooth concordance group to $\mathbb{Z}$ and it is a lower bound for the slice genus. In fact, s determines the slice genus for torus knots $T_{p,q}$, which was previously only accessible via gauge theory.

In 2005, Mackaay, Turner, and Vaz generalized Rasmussen's construction, which used rational coefficients, and found analogous invariants $s^{\mathbb{F}}$ for any choice of ground field $\mathbb{F}$ [5]. Seed showed that $s^{\mathbb{Q}}$ and $s^{\mathbb{F}_2}$ differ—a specific example is given by the knot $J = 14n19265$ [4, 8]. But what about other fields? When p is an odd prime, is it possible that the $s^{\mathbb{F}_p}$ agree with the original invariant $s = s^{\mathbb{Q}}$?

The knot K on the front of the card is the 8-twisted positive Whitehead double of $T_{3,4}$, denoted $D_+(T_{3,4}, 8)$; two independent programs confirm that $s^{\mathbb{Q}}(K) \neq s^{\mathbb{F}_3}(K)$ [3, 7]. Intriguingly, Whitehead doubles, such as $D_+(T_{2,3}, 2)$, were also the first known examples for which $s^{\mathbb{Q}}$ differs from the Ozsváth-Szabó concordance invariant from knot Floer homology, which is gauge-theoretic in nature [1]. By comparing the Rasmussen invariants for K and J, we see:

Theorem. *The Rasmussen invariants $s^{\mathbb{Q}}$, $s^{\mathbb{F}_2}$, and $s^{\mathbb{F}_3}$ are linearly independent as homomorphisms from the smooth concordance group.*

The right-hand side of the card indicates how the Rasmussen invariants are computed, using the multicurve techniques of [2], from a decomposition of K into the tangles T_1 and T_2. The non-compact component of the invariant $\widetilde{\mathrm{BN}}(T_2)$ has a different slope over $\mathbb{F}_3$ than over $\mathbb{F}_p$ for primes $p \neq 3$. (These invariants were computed with [9] using $\mathbb{F}_p$ for large p as an approximation for $\mathbb{Q}$.) The Rasmussen invariants can be read off from the quantum gradings of the highlighted intersection points of the blue curves with the red curve, which is the invariant $\widetilde{\mathrm{BN}}(T_1)$. This approach may be used to construct an interesting infinite family of knots for which $s^{\mathbb{Q}} \neq s^{\mathbb{F}_3}$; this is the subject of a forthcoming article.

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