

schedule

Perspectives on Quantum Link Homology Theories

Venue: Hörsaal H32 (UR Mathematik, 1. OG)

Monday	15.00	coffee & tea & registration	}	workshop
	15.45	lecture 1 Lobb		
	16.45	break		
	17.00	lecture 1 Zhang*		
Tue/Wed/Thu	10.30	coffee & tea		
	11.00	lecture 1/2/3 Wedrich		
	12.00	lunch		
	13.15	lecture 1/2/3 Robert		
	14.15	Q&A/exercise session		
	15.15	break		
	15.45	lecture 2/3/4 Lobb		
	16.45	break		
	17.00	lecture 2/3/4 Zhang*		
Friday	10.30	coffee & tea		
	11.00	lecture 4 Wedrich		
	12.00	lunch		
	13.15	lecture 4 Robert		
	14.15	Q&A/exercise session		
	15.15	break		
	15.30	special lecture Rasmussen		
	16.30	reception		
Saturday	10.00	coffee & tea	}	conference
	10.30	talk Marengon*		
	11.20	break		
	11.50	talk Anghel*		
	12.40	lunch		
	14.00	talk Kotelskiy*		
	14.50	break		
	15.20	talk Watson*		
16.10	talk Alishahi*			
Sunday	10.00	coffee & tea		
	10.30	talk Putyra*		
	11.20	break		
	11.50	talk Queffelec*		

* speaker will join virtually

workshop programme

Perspectives on Quantum Link Homology Theories

Lobb: Spectral Sequences and Khovanov homology

Monday–Thursday 15.45

—abstract not yet available—

Zhang*: A perspective on annular Khovanov homology

Monday–Thursday 17.00

A link L living in a thickened annulus $(\mathbb{R}^2 \setminus \{0\}) \times [0, 1]$ is called an annular link. The ambient space equips the bigraded Khovanov chain complex of L with an additional annular (filtration) grading, and the associated graded complex computes the annular Khovanov homology, $\text{AKh}(L)$. Understanding annular knots and links is important in many different contexts in low-dimensional topology; this lecture series will survey some existing and potential relationships between AKh and contact topology, knot concordance, representation theory, and more.

Some familiarity with Khovanov homology at the level of Dror Bar-Natan’s introductory paper on Khovanov homology (ArXiv: math/0201043) is recommended. Fun computational exercises will be provided.

Wedrich: Invariants of 4-manifolds from Khovanov-Rozansky link homology

Tuesday–Friday 11.00

Ribbon categories are 3-dimensional algebraic structures that control quantum link polynomials and that give rise to 3-manifold invariants known as skein modules. I will describe how to use Khovanov-Rozansky link homology, a categorification of the $\mathfrak{gl}(N)$ quantum link polynomial, to obtain a 4-dimensional algebraic structure that gives rise to vector space-valued invariants of smooth 4-manifolds, following ArXiv: 1907.12194 with Scott Morrison and Kevin Walker.

1. Khovanov-Rozansky $\mathfrak{gl}(N)$ link homology: introduction, basic properties, functoriality
2. The categorified Kauffman trick and its applications
3. Skein modules for 4-manifolds and computational methods
4. Towards an extended TQFT

Robert: $\mathfrak{gl}(N)$ link homology via foams

Tuesday–Friday 13.15

The aim of these lectures is to give a down-to-earth introduction to Khovanov–Rozansky colored $\mathfrak{gl}(N)$ link homology theories which categorify colored quantum $\mathfrak{gl}(N)$ link polynomials. This approach uses foams and is as close as possible to Bar-Natan’s definition of Khovanov homology.

1. I'll give a definition of colored $\mathfrak{gl}(N)$ quantum link invariants using graph colorings and sketch a proof of invariance. I'll use an interpretation of quantum binomials as graded cardinals.
2. Combinatorial ideas of Lecture 1 are upgraded to a 2 dimensional setting: We will consider foams instead of graphs. I'll give foam evaluation formulas and show that they satisfy some local relations.
3. I'll explain how to derive functors from foam evaluations and how to use it to construct colored $\mathfrak{gl}(N)$ link homology theories. I'll focus on the uncolored case and I'll sketch proof of invariance.
4. I'll give an overview of some other link homology theories called symmetric link homology. They use the same foams evaluation technique but in an annular setting.

conference programme

Perspectives on Quantum Link Homology Theories

Rasmussen*: title tbc

Friday 15.30

—abstract not yet available—

Marengon*: A generalization of Rasmussen’s invariant, with applications to surfaces in some four-manifolds

Saturday 10.30

We extend the definition of Khovanov-Lee homology to links in connected sums of $S^1 \times S^2$, and construct a Rasmussen-type invariant for null-homologous links in these manifolds. For certain links in $S^1 \times S^2$, we compute the invariant by reinterpreting it in terms of Hochschild homology. As applications, we prove inequalities relating the Rasmussen-type invariant to the genus of surfaces with boundary in the following four-manifolds: $B^2 \times S^2$, $S^1 \times B^3$, $\mathbb{C}P^2$, and various connected sums and boundary sums of these. We deduce that Rasmussen’s invariant also gives genus bounds for surfaces inside homotopy four-balls obtained from B^4 by a certain operation called Gluck twists. Therefore, Rasmussen’s invariant cannot be used to prove that such homotopy four-balls are non-standard.

Anghel*: Coloured Jones and Alexander polynomials unified through Lagrangian intersections in configuration spaces

Saturday 11.50

The theory of quantum invariants started with the Jones polynomial and continued with the Reshetikhin-Turaev algebraic construction of link invariants. In this context, the quantum group $U_q(\mathfrak{sl}(2))$ leads to the sequence of coloured Jones polynomials, which contains the original Jones polynomial. Dually, the quantum group at roots of unity gives the sequence of coloured Alexander polynomials. We construct a unified topological model for these two sequences of quantum invariants. More specifically, we define certain homology classes given by Lagrangian submanifolds in configuration spaces. Then, we prove that the N th coloured Jones and N th coloured Alexander invariants come as different specialisations of a state sum (defined over 3 variables) of Lagrangian intersections in configuration spaces. As a particular case, we see both Jones and Alexander polynomials from the same intersection pairing in a configuration space.

Kotelskiy*: Khovanov homology via Floer theory of the 4-punctured sphere

Saturday 14.00

Consider a Conway two-sphere S intersecting a knot K in 4 points, and thus decomposing the knot into two 4-ended tangles, T and T' . We will first interpret Khovanov homology $\text{Kh}(K)$ as Lagrangian Floer homology of a pair of specifically constructed immersed curves $\text{Kh}(T)$ and $\text{BN}(T')$ on the dividing 4-punctured sphere S . Next, motivated by tangle-replacement

questions in knot theory, we will describe a recently obtained structural result concerning the curve invariant $\text{Kh}(T)$, which severely restricts the types of curves that may appear as tangle invariants. The proof relies on the matrix factorization framework of Khovanov-Rozansky, as well as the homological mirror symmetry statement for the 3-punctured sphere. This is joint work with Liam Watson and Claudius Zibrowius.

Watson*: title tbc

Saturday 15.20

—abstract not yet available—

Alishahi*: Braid invariant related to knot Floer homology and Khovanov homology

Saturday 16.10

Knot Floer homology and Khovanov homology are homological knot invariants that are defined using very different methods — the former is a Lagrangian Floer homology, while the latter has roots in representation theory. Despite these differences, the two theories contain a great deal of the same information and were conjectured by Rasmussen to be related by a spectral sequence. This conjecture was recently proved by Dowlin, however, his proof is not computationally effective. In this talk we will sketch a local framework for proving this conjecture. To do that, we will describe an algebraic/combinatorial glueable braid invariant which using a specific closing up operation results in a knot invariant related by a spectral sequence to Khovanov homology. Moreover, it is chain homotopic to Ozsvath-Szabo's braid invariants which using their closing up operation recovers knot Floer homology. If time permits we will compare the two closing up operations. This is a joint work with Nathan Dowlin.

Putyra*: title tbc

Sunday 10.30

—abstract not yet available—

Queffelec*: Surface skein algebras, categorification and positivity

Sunday 11.50

Skein algebra for surfaces are natural generalizations of the Jones polynomial to thickened surfaces. Khovanov homology can be extended beyond the 3-sphere following a similar process, but the algebra structure is trickier to understand at the categorical level, partly because of the lack of functoriality of Khovanov's original construction. I'll review ways to understand the skein category of a surface, and explain how we're trying to use these tools to prove a conjecture by Fock-Goncharov-Thurston claiming that the skein algebras have positive structure constants.

This is joint work with Kevin Walker and Paul Wedrich