A generalization of Riemann's inequality, with applications to surfaces in some 3-manifolds

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§1. A reminder of topology

LEMMA + a bonus. Let M be a closed, connected, 3-dimensional M^3, g be a Riemannian metric.

1. Let $\gamma \hookrightarrow (M, g)$ be a closed, connected, smooth curve. Then $\exists$ a sequence of γ $\{\gamma_i\}_{i\in\mathbb{N}}$ such that $\lim_{i\to\infty} \mathrm{Area}(\gamma_i) = \mathrm{Area}(\gamma)$ and $\lim_{i\to\infty} \mathrm{Area}(\gamma_i) = \mathrm{Area}(\gamma)$.

2. $\mathrm{Area}(\gamma) \geq \mathrm{Area}(\gamma_i)$ $\forall i \in \mathbb{N}$ (minimal area of γ).

Let $\mathrm{Area}(\gamma) \leq \mathrm{Area}(\gamma_i)$ $\forall i \in \mathbb{N}$ (minimal area of γ).

Then $\mathrm{Area}(\gamma) =