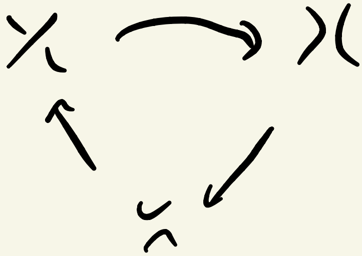
Kh can be thought of as the combinatorial part of homologies satisfying a skein rela.



L.E.S.

First eg. Oz-Sz's spec_seq. over $\mathbb{F} = \mathbb{Z}/2$

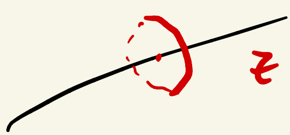
$$E_2 = Kh_{red}(K) \longrightarrow \widehat{HF}(\Sigma(K)) = E_\infty$$

chain groups have $\frac{1}{2}$ the dim of usual chain groups.

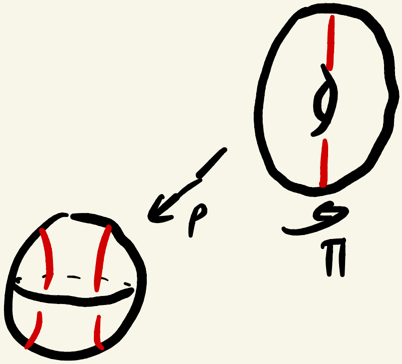
branched double cover of K.

$$\Sigma(K) = \begin{array}{c} M^3 \\ \big\downarrow \times 2 \\ S^3 \setminus K \end{array} \cup K$$

$$\begin{array}{c} \Sigma(K) \\ \big\downarrow \\ S^3 \end{array} \quad \text{2-to-1 except at } K$$



$z \mapsto z^2$



$)(\,,\ \asymp\,,\ \times$

$\Sigma()(),\ \Sigma(\asymp),\ \Sigma(\times).$

These are each filling of some 3-manifold w/ torus boundary.

Floer homologies satisfy a surgery exact sequence

$$\begin{tikzcd}
\widehat{HF}(\Sigma(\times)) \arrow[rr] & & \widehat{HF}(\Sigma()()) \arrow[dl] \\
& \widehat{HF}(\Sigma(\asymp)) \arrow[ul] &
\end{tikzcd}$$

L.E.S.

Figure out what is

$$\widehat{HF}(\Sigma(\text{n-component unlink}))$$

and figure out

$$\begin{tikzcd} \widehat{HF}(\Sigma(00)) \arrow[r, bend left] & \widehat{HF}(\Sigma(0)) \arrow[l, bend left] \end{tikzcd}$$

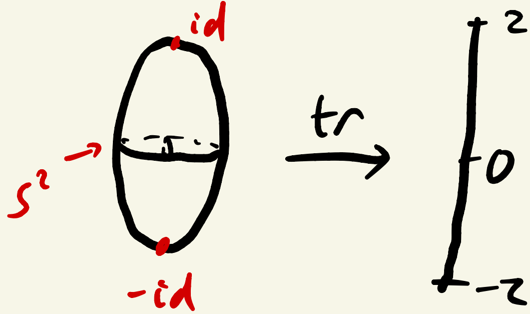
one sees that these mimic the (reduced) Khovanov chain complex.

Instanton knot Floer homology (over $\mathbb{Z}$)
(Kronheimer-Mrowka).

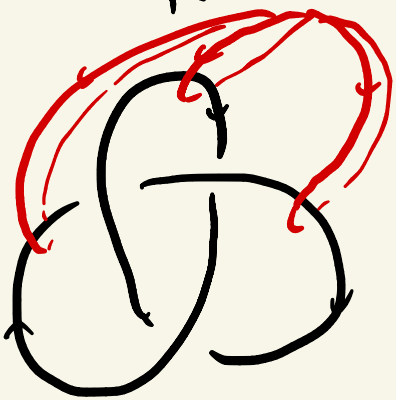
$$Kh \rightsquigarrow I^*$$

Chain groups of $I^*(K)$ are (roughly speaking) given by the homology of a space of flat $SU(2)$ connections on $S^3 \setminus K$:

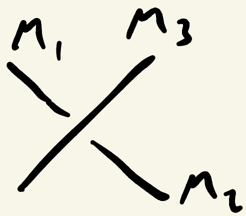
$SU(2) \cong S^3$

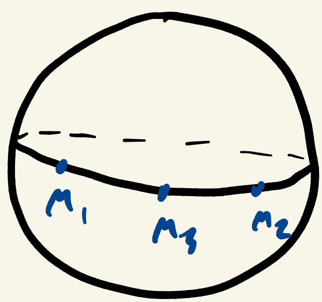


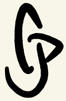
$$\mathrm{Rep}(K) = \{\rho : \pi_1(S^3 \setminus K) \to SU(2) : \mathrm{tr}(\rho(m)) = 0\}.$$



To specify a ρ we give a pt on S^2 for each meridian to the arcs of a knot diagram.

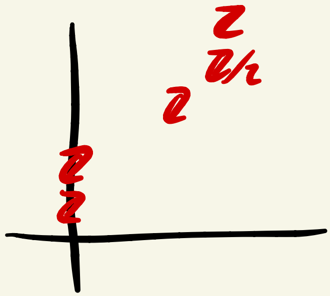




Rep()

	S^2	$UTS^2 \cong \mathbb{RP}^3$
H_*	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}/2$

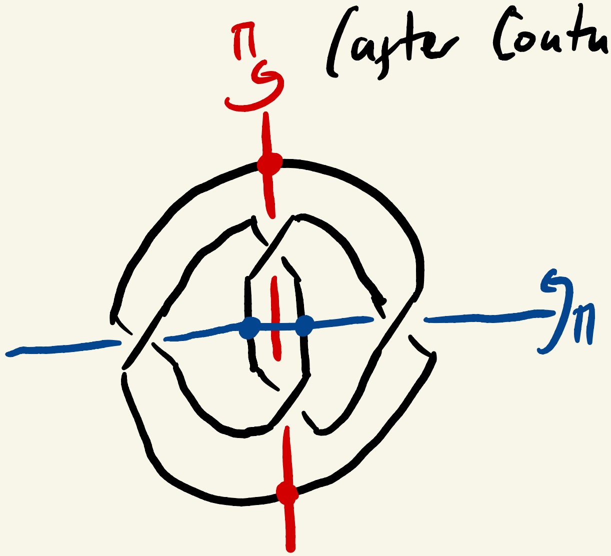
$Kh^{i,j}$()



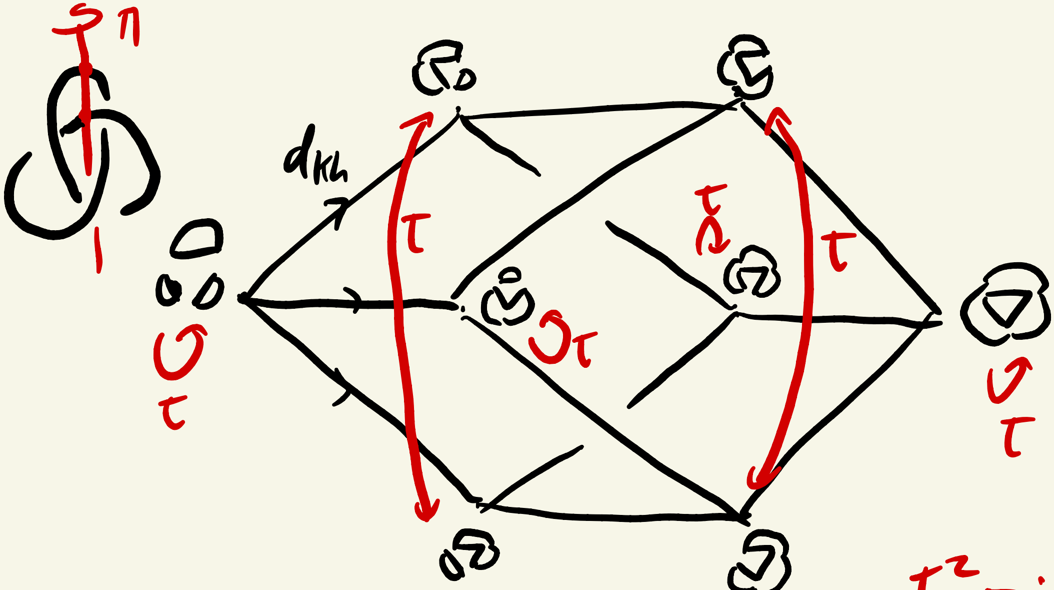
See work of Lewallen.

Esoteric eg (L. - Watson)
(after Couture).

Over $\mathbb{F} = \mathbb{Z}/2$.



Knots w/ "strong involution" (rotational sym. w/ 2 fixed pts).



$\tau^2 = id$.

So $d_\tau := id + \tau$ is a differential

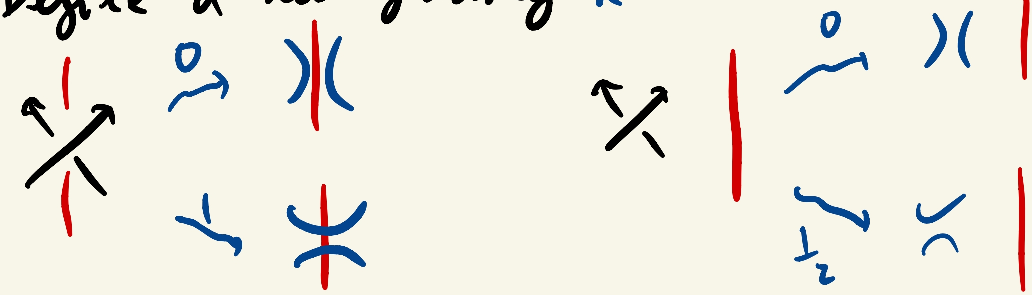
$d = d_{Kh} + d_\tau$ is also a differential (exercise).

Call (C, d) the complex w/ homology H.

d preserves j, changes i by less than or equal to 1.

Get a filtration $\mathcal{F}^i C^j(D)$ w/ E_2 page $E_2 = Kh^{i,j}$.

Define a new grading k



This grading is preserved by d_T, raised by d_{Kh}. Create a filtration $\mathcal{G}^k C^j$ w/ first differential in spec. seq. d_T. The E_3 page and later are invariants.

Double filtered, j-graded homology.

Triple graded associated graded homology

$$Gr^i Gr^k H^j(K).$$

Triple grading can be used (e.g.) to distinguish strong inversions on the same knot.