$$D \rightsquigarrow CKh^{i,j}(D) \circlearrowleft \partial$$

(ort'd). $\quad i,j \in \mathbb{Z}$ $\quad \partial^2 = 0$

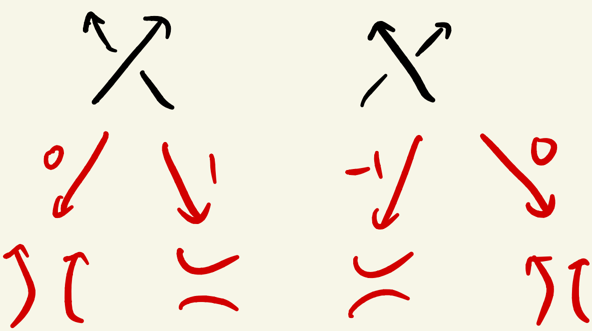
fin. gen. free abelian complex

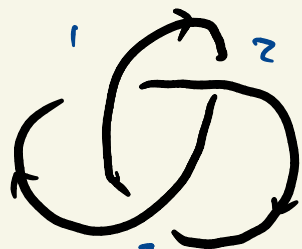
$$\partial : CKh^{i,j}(D) \to CKh^{i+1,j}(D)$$

Take cohomology get

$Kh^{i,j}(K)$. $\quad$ D is diagram of knot K.

+'ve $\qquad$ -'ve





$n_+ = 0$

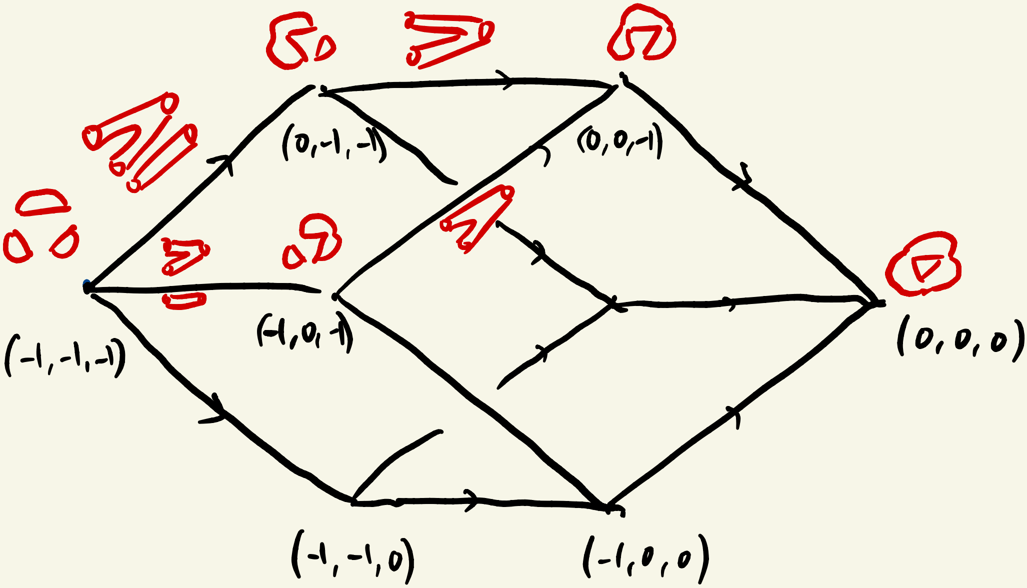
$n_- = 3$

$n = n_+ + n_- = 3$

$w = n_+ - n_- = -3$.

Decorate cube

$[-1,0]^{n_-} \times [0,1]^{n_+}$ by putting smoothings at each vertex.



Q: In what sense is this already a knot invariant? (Bar-Natan).

We shall apply some algebra to land in the category of bigraded cochain complexes. $[CKh^{i,\delta}(D)]$

Apply a 1+1 dim'l TQFT to the vertices and edges.

1-manifolds + cobordisms $\xrightarrow{\text{TQFT}}$ Abelian groups + homomorphisms.

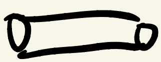
$$\coprod^r \bigcirc \rightsquigarrow \langle v_-, v_+ \rangle^{\otimes r}$$

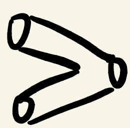
generators are r-strings of +'s and -'s.

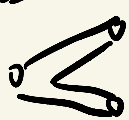
$$v_- \otimes v_+ \otimes v_- \in \langle v_+, v_- \rangle^{\otimes 3}$$

or decorations of $\coprod^r \bigcirc$ with +'s and -'s.

$$V = \langle v_-, v_+ \rangle$$

 $\rightsquigarrow V \xrightarrow{id} V$

 $\rightsquigarrow V \otimes V \xrightarrow{m} V$

 $\rightsquigarrow V \xrightarrow{\Delta} V \otimes V$

 $V \to \mathbb{Z}$

 $\mathbb{Z} \to V$

$m : V \otimes V \to V$

$v_+ \otimes v_+ \mapsto v_+$

$v_+ \otimes v_- \mapsto v_-$

$v_- \otimes v_+ \mapsto v_-$

$v_- \otimes v_- \mapsto 0$

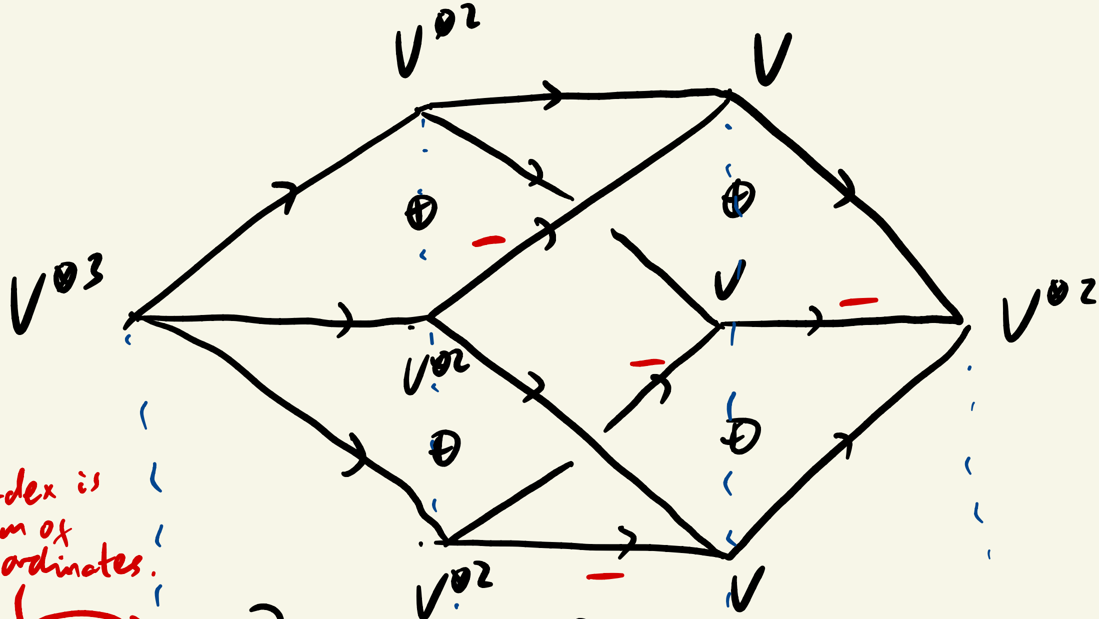
$\Delta : V \to V \otimes V$

$v_+ \mapsto v_- \otimes v_+ + v_+ \otimes v_-$

$v_- \mapsto v_- \otimes v_-$

m, Δ each drop "degree" by 1.

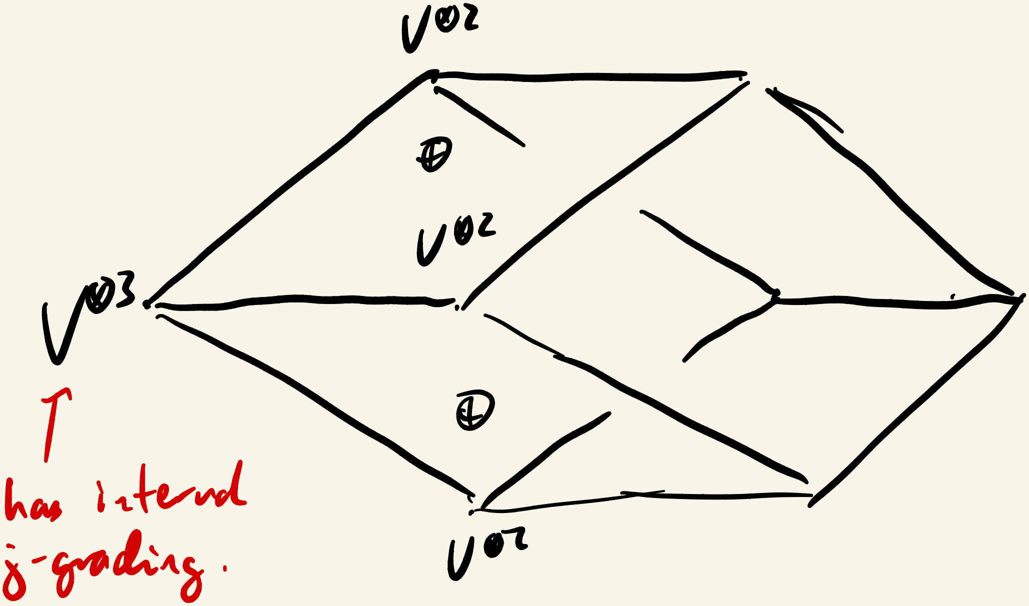
Sum of +'s and −'s.



Index is sum of coordinates.

$$C^{-3} \xrightarrow{\partial} C^{-2} \xrightarrow{\partial} C^{-1} \xrightarrow{\partial} C^{0}$$

Note that each square (anti-) commutes



The j-grading (quantum grading) is preserved by ∂. It arises from the "degree" we mentioned earlier (sum of +'s and −'s).

$$C^i = \left(\bigoplus_{j=1}^{r} V^{\otimes p_j} \right) [\underbrace{i}_{\text{cohomological degree}} + \underbrace{w}_{\text{writhe}}]$$

$$C^i = \bigoplus_{j \in \mathbb{Z}} C^{i,j}$$

$$\partial : C^{i,j} \longrightarrow C^{i+1,j}$$

This is the complex $CKh^{i,j}(D)$.

① Why is the cohomology invrt under Reidemeister moves?

② What is χ of $Kh^{i,j}(K)$?

③ What about functoriality?